Electric Vehicle Enhanced Range, Lifetime And Safety Through INGenious battery management

D3.5 – Report on driving range prediction and extension algorithm
February 2020

This project has received funding from the European Union’s Horizon 2020 research and innovation programme under grant agreement No 713771
# D3.5 – Report on driving range prediction and extension algorithm

Author: Paul Padilla (TU/e)

EVERLASTING - Grant Agreement 71377 (Call: H2020-GV8-2015)
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| **Author(s)**   | Paul Padilla (TU/e)  
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| **Reviewer(s)** | Will Hendrix (TUE)  
|                  | Roshni Digumoorthi (VDL)  
|                  | Juan Carlos Flores (VDL)                                             |
| **Type**        | Report                                                              |
| **Dissemination level** | PUBLIC                                                             |
| **Due Date**    | M42                                                                 |
| **Submission date** | January 27, 2020                                                   |
| **Status and Version** | Draft, version 0.4                                                  |
## Revision History

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<td>20/01/2020</td>
<td>Paul Padilla (TUE)</td>
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<td>Tijs Donkers (TUE)</td>
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<td>Will Hendrix (TUE)</td>
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<td>V0.3</td>
<td>30/01/2020</td>
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<td>Juan Carlos Flores (VDL)</td>
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<tr>
<td>V0.4</td>
<td>04/02/2020</td>
<td>Carlo Mol (VITO)</td>
<td>Quality check</td>
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<tr>
<td>V1.0</td>
<td>DD/MM/YYYY</td>
<td>Carlo Mol (VITO)</td>
<td>Submission to the EC</td>
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ACKNOWLEDGEMENT

This project has received funding from the European Union’s Horizon 2020 research and innovation programme under grant agreement No 713771
EXECUTIVE SUMMARY

Range extension of electric vehicles has become a prominent objective to achieve in order to mitigate range anxiety and consequently, accelerate the adoption of electric vehicles in the market. In fact, range anxiety mitigation by vehicle range optimization has become one of the main objectives in the EVERLASTING project. Range optimization can be achieved by using vehicle energy management strategies, which aim to optimize the total energy consumption of the vehicle. In most of the cases, it is simple and economical compared to other approaches to extend the vehicle range, e.g., energy dense batteries, extended charging infrastructure, light weight vehicle parts, etc.

For this reason, this report documents the work done in Task 3.5, where the objective has been to optimize and to predict the driving range in terms of energy. The compendium of publications obtained in this project ([1]–[11]) are included as appendices of this report. In particular, the work carried out in Task 3.5 has lead to 2 journal publications, 7 refereed conference publications, 1 MSc thesis and 1 whitepaper. Furthermore, a preliminary experimental study shows that optimization of the vehicle energy consumption can lead to a reduction in energy consumption of 6.94%, which leads to a range extension of 7.45%, thereby laying the basis for achieving the objective of range extension of 5% as described in the project’s description of action (DoA).
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LIST OF ABBREVIATIONS AND ACRONYMS

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<td>Description of activity</td>
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<td>Complete Vehicle Energy Management</td>
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<td>HVAC</td>
<td>Heat Ventilation and Air Conditioning System</td>
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<td>Forward Wheel Drive</td>
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1 Introduction

This report documents the work done in Task 3.5. It contains a description of the main objectives and contributions of the work. The publications that have resulted from the work done in Task 3.5 are included in the appendix. This chapter connects the objectives of Task 3.5 to these publications.

An extensive literature review of the state-of-the-art exploration of the research paths on complete vehicle energy management for electromobility applications is presented in [1]. This exploration is directed to the modelling, optimization techniques and prediction in view of exploitation of preview information aspects to predict power request that arise from the inclusion of velocity as a decision variable in energy management problems. Thus, aiming to include ‘anticipation’ and ‘reaction’ as way of predictions as new features of the complete vehicle energy management concept (CVEM).

In this work, energy management strategies are divided in 2 groups. First, CVEM that aims to maximize the range of the vehicle by minimizing the total power consumption using an optimal splitting of energy among all the interconnected subsystems in the vehicle for a given power request. Second, Eco-driving strategies, which extends the driving range of a vehicle by the selection of an energy optimal velocity profile.

A first contribution to a power request and range predictor has been described in [2]. Accurate range estimation in electric vehicles is a crucial challenge to increase customer trust in the use of batteries as an alternative source of power in mobility, thereby mitigating the effect of range anxiety for drivers. This work presents a novel approach to estimate the range of an electric vehicle with a low computational load by estimating the battery pack state and forecasting the future power demand using available vehicle information. The results show that the driving range of an electric bus can be accurately estimated with a maximum range estimation error of less than 10%. Range prediction can also be used as a tool to predict future power request in a vehicle, which is important aspect to consider during the implementation of CVEM strategies. In fact, the efficiency achieved by CVEM strategies depends on the quality of the predictions for future power request. Further explorations for more accurate predictions of the vehicle are performed in Task 3.3, whose results are reported in Deliverable 3.3.

In [5], an energy management strategy to improve the operational efficiency of a vehicle, and thus , extending the driving range is proposed as a distributed optimization approach is proposed to solve the CVEM problem of a vehicle with several controllable auxiliaries. This approach uses a dual decomposition, which allows the underlying optimal control problem to be solved for every subsystem separately. Simulation results show that the energy consumption can be reduced up to 0.52% on a heavy-duty truck, by including smart auxiliaries in the energy management problem. More interestingly, the computation time is reduced by a factor of 64 up to 1825, compared with solving a centralized convex optimization problem.

In [6], the methodology presented in [5] has been extended to include charge acceptance limitations and battery aging effects, showing that battery aging constraints can be partly compensated by smart control of other energy buffers, which shows the true benefit of CVEM.

The existence of only global optimal solutions for the CVEM problem was demonstrated in [7]. As a result, a distributed optimization algorithm for non-convex optimization problems using operator splitting techniques was proposed. The relevance of this contribution is that it opens the door for online implementations of CVEM using a decentralized architecture.

In [8], a different exploration of CVEM is performed using port-based models. Specifically, Port-Hamiltonian theory is used to formulate a decomposable optimal control problem for CVEM applications. The advantage of this alternative modeling framework with respect to the power-based approach used in [5],[6],[7],[9], is that it is able to capture physically relevant variables of the systems and describe energy loses in an intuitive way, which is relevant for this type of energy
 management problems. A simulation case study for an electric city bus with heating, ventilation and air conditioning (HVAC) system was presented, where it was possible to see that the HVAC system can be used as an additional energy buffer. The simulation results showed that in specific cases an extension of the driving range up to 25% could be possible.

Real driving conditions are subject to uncertainty, which directly affect power request predictions, thereby making the real-time optimization of the energy consumption of a vehicle to be a challenging problem. In order to deal with this situation, in [9] the current framework of CVEM was employed in a receding horizon fashion in which random constraints representing realizations of the disturbance, i.e., the uncertain driving conditions, are considered. Additionally, several methods for power request prediction are described and compared, e.g., using average traffic flow information, a method based on Gaussian process regression, and a method that combines both. The proposed strategies are tested with real traffic data as a case study where the use of a Gaussian process regression and the average traffic speed achieves near optimal fuel consumption.

The driving range of a vehicle can also be extended by the selection of an energy optimal velocity profile, which is referred as the eco-driving problem. Eco-driving is an energy management method that aims to maximize the range of a vehicle by means of finding energy optimal velocity profiles. In [3], a detailed analysis on the global optimal solution of the Eco-driving problem was presented. This result was exploited to propose a static non-linear optimization algorithm that can be used for an online implementation of this method. As a consequence of the results in [4], an online implementation of the eco-driving algorithm for electric city buses was presented. The optimization algorithm was used to formulate a shrinking horizon model predictive control that provides real time feedback to the driver about the optimal velocity profile needed to arrive the next bus stop on a given time. In order to validate this approach, experiments on an electric city bus were performed obtaining as a result average energy savings of 6.94%. It should be noted that energy savings of 6.94% leads to an extension of the range by 7.45%. Namely, it holds that

$$\text{Vehicle range}[\text{km}] = \frac{\text{Energy available}[\text{kWh}] \times \text{vehicle speed} \left(\frac{\text{km}}{\text{h}}\right)}{\text{Energy consumption}[\text{kW}]}$$

And a reduction in energy consumption by a factor \((1-0.0694)\) leads to an extension of the range by a factor \(1/(1-0.0694) = 1.0745\).

In [11], the work proposed in [3] is extended by considering a model for eco-driving that captures cornering effects, which is especially relevant in urban scenarios. The proposed model purely relies on the geometric configuration of the vehicle and road. Moreover, it can be applied for vehicles with front wheel drive (FWD) or rear wheel drive (RWD). Simulation results for an electric vehicle executing cornering maneuvers showed an approximated improvement of 8% in energy savings with respect to traditional eco-driving strategies, especially in trajectories with large curvatures.

Finally, in [8] the CVEM and eco-driving strategies (presented in [3] and [5]) are connected and extended to incorporate velocity profiles as part of the CVEM problem. It is shown that a Sequential Quadratic Programming (SQP) algorithm, which uses a convex subproblem with Tikhonov regularization, can be embedded in a distributed optimization approach, allowing it to be used for CVEM, incorporating optimal control of the vehicles auxiliary systems, in combination with eco-driving.
2 Conclusion

In the Description of Activities (DoA), objective 6 states:

**Objective 6: To extend the driving range the EVERLASTING project will develop BMS technology that will increase driving range up to 15% without increasing the physical battery size. It will also decrease the charging time by 30%.**

To reach the 15% increase of driving range several topics have been listed in the DoA to be subject of research, where the results of this separate research activities all contribute to reach this 15% target.

This Deliverable focusses on the topic:

- *Optimizes the driving behavior, resulting in an increase of the driving range by 5%.*

The above discussed publications have explored range prediction, which crucially depends on the vehicle’s ability to predicts the vehicle’s power request. This is considered in more detail in Task 3.3 and Task 3.4, with corresponding Deliverables 3.3 and 3.4. Furthermore, algorithms that can be used to extending the driving range have been studied extensively and first experimental results show that an extension of the driving range of more than 5% is expected, as was intended as one of the project’s overall objectives.

3 List of Publications


Complete Vehicle Energy Management for Electromobility

Paul Padilla

Abstract—This paper presents an exploration of the future research on complete vehicle energy management for electromobility applications. This exploration is directed to the modelling, optimization techniques and exploitation of preview information aspects that arise from the inclusion of velocity as a decision variable in energy management problems. Thus, aiming to include ‘anticipation’ and ‘reaction’ as new features of the complete vehicle energy management concept. Preliminary results that show the potential of this research topic are presented and concrete plans for future extensions are discussed.

I. INTRODUCTION

Transportation has deeply influenced the evolution of our civilization. The formation of modern urban centers has been shaped by the development of transportation systems, which is also directly connected to the economic welfare; e.g., transporting raw materials and finalized goods over long distances is an essential activity for trading. On the other hand, the environmental role of transportation has a negative impact for the modern society. For instance, the transport sector was responsible for 14% of the global CO₂ emissions from fossil fuel consumption in 2014 [1]. Additionally, in 2016, it was estimated that 95% of the global transportation energy was based on petroleum fuels [2]. This oil dependency is also a sensitive factor that influences the geopolitical stability since petroleum is a finite resource.

Despite of those negative factors, transportation still encompasses crucial aspects in our society. Thus, in the last years, the electrification of transport systems has raised as a promising approach to mitigate effects caused by the environmental role of transportation and to face the imminent depletion of fossil fuels. Assuming that the electricity consumed by electric transportation systems is produced from renewable energy sources, the impact on the reduction of CO₂ emissions is direct; under the same assumption, it is expected that electric transportation drastically reduce the high oil dependency of the modern society [3].

The electrification in automotive sector has been studied as electromobility in [4], which can be defined as vehicles that are propelled by electricity. This is a simple yet broad definition due to the fact that not only encircles hybrid electric vehicles (HEV) or battery electric vehicles (EV), this definition also includes vehicles that obtain energy form the grid or converts other energy sources to electricity, i.e., biomass, hydrogen, etc. Hence, electromobility implies flexibility for the electrified vehicles to have different energy sources [4].

The evolution of the EV market in the recent years have opened the door for optimistic predictions about the future market for electromobility. For instance, in 2016, the EV made up 29% of the market in Norway, while in China, the number of EV units increased from 100,000 in 2014 to 650,000 in 2016 [5]. This tendency indicates that by 2030 electromobility might reach a stage of mass market adoption. This global optimism is also supported by the novel research announcements related to batteries, charging infrastructure and energy efficiency in vehicles.

However, the goal of a complete acceptance of electromobility in the market is still far and several problems need to be solved. It has been identified that one of the main reasons that limits the penetration rate of EV in the market is range anxiety [6], [7]; which is defined as the concern (experienced by users) that the vehicle has insufficient energy to reach the next charging station [5]. There are several factors that influence range anxiety, e.g., reduced capacity of batteries to store energy, weather and traffic conditions that affect drastically the prediction of the remaining driving range [8], limited number of charging stations and the time-consuming charge process [9]. As a response to these factors, the mitigation of the anxiety range can be achieved by a combination of the next approaches:

1) Energy management strategies.
2) Light weight and energy dense batteries.
3) Fast charging.
4) Extended charging infrastructure.

The use of light-weight and energy dense batteries is the most logical choice to reduce the energy anxiety effect by extending the driving range of the EV. On the other hand, the extension of charging infrastructure and the development of fast charging technologies aim to reduce the range anxiety by offering the user the possibility to always have a reachable charging station where the charging process is performed in a reduced time, which is resembles the functionality of the current gas stations. The last proposed approach to mitigate range anxiety is related to energy management strategies, which aims at extending the driving range of the vehicle by maximizing the energy efficiency during its operation. In particular, energy efficient operation of vehicles can be described as the solution to an optimal control problem that aims to obtain the optimal conversion of energy between the energy consumers in the power network of the vehicle. The main advantage for this approach is that energy management strategies can (for most of the cases) be applied directly as software in the vehicle. Hence, the implementation cost is marginal. For this reason, energy management strategies have been a topic of intensive research in the last years and defines the global scope of the contents treated in this manuscript.

This paper aims to present a future view of energy management strategies for electromobility.

The content in this document is organized as follows. In
Section II, an overview of architectures and basic modeling for energy management applications are presented. The state of the art for energy management systems is discussed in Section III. In Section IV, the main research question is posed, several problems are formulated and possible methodologies to solve those problems are discussed. Preliminary results are presented in Section V. Finally, the future work and conclusions are discussed in Section VI and Section VII respectively.

II. OVERVIEW OF ENERGY MANAGEMENT CONCEPTS

In this section, the concept of complete vehicle energy management (CVEM) is presented. Additionally, an overview of the power network concepts is presented as a general network architecture applied to electromobility.

A. Complete Vehicle Energy Management

The CVEM concept was originally introduced in [10] as an holistic approach to energy management in vehicles. Later, methodologies to solve the CVEM problem in several automotive applications have been reported in [11]–[13]. The individual optimization of energy consumption in all the subsystems of a vehicle is incapable to produce a global energy efficient operation. This limitation is consequence of neglecting the iteration that exists between all the energy consumers inside a vehicle. Hence, an holistic approach that optimizes energy consumption of all the interconnected subsystems in the vehicle is needed, which is the basis for the CVEM concept.

The energy in the vehicle is transformed between different physical domains, e.g., chemical energy that is stored as fuel can transformed into electrical energy by an electric generator unit; later, the produced electricity can be either transformed into mechanical energy to propel the vehicle or transformed into thermal energy modify the temperature of the cabin. For this reason, considering power networks to describe the power iteration between the energy consumers inside a vehicle is feasible. Hence, an holistic approach that optimizes energy consumption of all the interconnected subsystems in the vehicle is needed, which is the basis for the CVEM concept.

By considering the iteration of all the subsystems in the power network, the design of energy management strategies that determine a globally efficient operation of the vehicle is feasible.

B. Power Network Architecture for Electromobility

Due to the broad nature of electromobility it is important to have a general architecture that can describe a large variety of vehicle topologies.

C. CVEM as an Optimal Control Problem

The CVEM problem can be described as an optimal control problem. The formulation of the problem depends
on the application goals; however, it is possible to identify a general structure that CVEM follows and is given by
\[
\min_{\xi(\tau)} J(\xi(\tau)) \\
\text{subject to} \quad D(\xi(\tau)) = 0, \quad (1b) \\
H(\xi(\tau)) = 0, \quad (1c) \\
N(\xi(\tau)) = 0, \quad (1d) \\
b(\xi(\tau)) \leq R(\xi(\tau)) \leq b(\xi(\tau)), \quad (1e)
\]
where \(\xi(\tau)\) is a time or space dependent vector of decision variables that normally represent power, forces or even velocities in the vehicle. The cost function of the optimal control problem \(1)\) is given by \(J(\xi(\tau))\), which it is normally described as the cumulative power consumed from the energy sources in the vehicle over a driving cycle. The problem is subject to the dynamics of all the subsystem \((1b)\), the power definition for each converter in the network \((1c)\) and to \((1d)\) that describes the power balance for all the nodes in the power network. Additionally, the optimal control problem \(1)\) is bounded by \((1e)\), which often represents power, torque and even velocity bounds.

III. STATE OF THE ART

A large amount of energy management strategies (EMS) have been reported in the last two decades. As can be seen in Fig. 2, the number of papers published in 2017 is approximately two times larger than the amount of papers published in 2010; thus, showing the growing interest of EMS the automotive industry. In order to describe the state of the art in this field, in this paper, the research literature available is divided in four relevant groups that can be directly connected to electromobility.

A. Classic EMS

Traditionally, energy management problems are focused on controlling the power split between the combustion engine and the electric machine of a hybrid electric vehicle. By storing regenerative braking energy and shifting the operating points of the combustion engine, a significant amount of fuel can be saved. In [14], [15, Ch. 4] optimization techniques for energy control on hybrid vehicles are summarized; additionally, the books [16], [17] present a complete introduction of this research area.

Classic EMS literature can be classified in an ad hoc solutions and optimal control approaches. Additionally, the optimal control approaches can be subdivided into off-line and on-line control problems.

1) Ad-hoc solutions:
The set of EMS that are included in this classification are normally characterised to be computationally fast, which make them suitable candidates for on-line implementations. Historically, this type of approaches were the first to appear in EMS literature [18], [19]. Fuzzy logic [20] and neuronal networks [21] are popular strategies among this classification. The disadvantage of this strategies is that the performance of the systems is sensitive to changes in operation, and the global optimality of the solution cannot be guaranteed. Nevertheless, some authors are currently proposing solutions to properly recalibrate this strategies using dynamic programming (DP) [22], thus obtaining an acceptable approximation of the global solution.

2) Off-line optimal control problems:
EMS under this category are normally used as benchmark solutions for specific applications or configurations of the optimal control problem due to the fact that that global optimal solutions are typically achieved.

A representative optimization technique in this category is Dynamic Programming (DP) [23]; for instance in [24], applying DP forces the convergence to the global optimum of the energy management problem proposed. However, DP has the inherent disadvantage that the computational burden increases with the number of states.

Optimization methods based on the Pontryagin’s Maximum Principle (PMP), see, e.g., [25], [26] can handle computational complexity of multi-state energy management problems. In PMP, the problem is reduced to solving a two-point boundary value problem, which can be difficult to solve in the presence of state constraints. Moreover, the global optimality of the solutions obtained can only be guaranteed if the formulation of the optimal control problem is convex.

Similarly, static optimization methods can guarantee a global optimality only for convex approximations of the energy management problems, e.g., see [27] and the references therein.

Finally, it is important to remark that the EMS that belong to this classification need to have a priori complete information related to driving cycle, i.e., velocity and road gradient. This is the main reason that limits the use of this EMS approach for on-line implementation.

3) On-line optimal control problems:
In this case, algorithms that requires a low computational effort are described; besides, the requirement of a priori
information related to the velocity and road profile is relaxed, which normally leads to a suboptimal solutions.

In [28], [29] stochastic DP is used to performance an off-line calculation of the optimal EMS, which is given by an stochastic policy that is implemented as a look-up table in a low computational power embedded system.

A fast computational approach that uses PMP is known as the equivalent consumption minimization strategy (ECMS) [30]. In this strategy, the energy consumption in the battery is transformed into an equivalent fuel consumption, which is represented by a co-state function related to the battery energy. The co-state can be estimated at every time instant, thus eliminating the necessity for the complete driving profile information in advance. Consequently, ECMS can be implemented on-line; however, updating the co-state is a delicate task that often produce to suboptimal solutions.

Model predictive control (MPC) is also used to implement on-line EMS [31], [32]. A finite-time horizon prediction of the future the energy consumption is used to calculate optimal control strategy, from which only the first control decision is implemented. A successive execution of this procedure obtains a suboptimal energy management policy that depending on the quality of the predictions can be very approximated to the optimal solution.

B. CVEM Approaches

In general, the classic EMS approaches consider only a reduced set of subsystems, i.e., the power interaction between the the internal combustion engine, electric machine and batteries. This motivated the emergence of the CVEM concept, as it was already discussed in Section II-A.

CVEM aims to extend classic EMS approaches to incorporate more subsystems in the optimal control problem. The goal is to increase scalability of the static optimization problem to allow for a large number of auxiliary systems. For instance, DP can provide global optimal solutions to CVEM problems, however, scalability becomes an issue due to the curse of dimensionality observed in this technique. Similarly, the inherit set of state constraints presented in the CVEM problem makes PMP approaches difficult to be used for this application. As a consequence, centralized and decentralized optimization approaches have been proposed to satisfy the requirements imposed by the CVEM concept.

The research conducted within [11] has made some early attempts to incorporate auxiliaries into the classic EMS approaches. Due to the use of a centralized optimization methods and convexification techniques the optimal control formulations ‘global optimality’ and ‘scalability’ requirements for CVEM are satisfied. However, the approaches reported are not flexible, i.e., the EMS is not easily reconfigurable when new subsystems are introduced in the optimization problem due to the convexification procedure. Additionally, some of the results presented in [11] have included the vehicle inertia in the energy management problem, which was solved by proposing a convex reformulation of the problem in space domain; in this case, the lack of flexibility is also an unsolved issue.

The research results presented in [12] uses game-theoretic approach to solve the CVEM problems in a decentralized manner, where all the subsystems share a limited amount of information and are able to take some decisions autonomously. On-line implementations were obtained within this approach, where predicted information is not considered, thus limiting the performance of the strategies. In this research, the inclusion of battery wear and charging acceptance are promising research directions that were not addressed. Moreover, an exploration of algorithms that converge to a Nash’s equilibrium are still an open task in this study.

A distributed optimization approach for CVEM is explored in [13] obtaining both on-line and off-line EMS implementations that are scalable and flexible for a large number of subsystems (including on/off auxiliaries) in the vehicle. This approach uses a dual decomposition to split a large and complex optimal control problem into several simple problems related to the subsystems in the vehicle. These sub-problems iteratively share information with a central coordinator in order to achieve an equilibrium that is represented by the optimal solution of the CVEM problem. The off-line formulations of the distributed optimization problem presented in [13] are able to drastically reduce the simulation time compared to classic EMS; however, this approach requires a priori information of the driving profile and power consumption. Furthermore, there are open questions related to the numerical aspects of the algorithms proposed to solve the distributed optimization problem, i.e., the convergence of the algorithms proposed has not been formally proved. On the other hand, the distributed optimization on-line formulation of the CVEM problem uses finite-time horizon predictions in an MPC fashion to obtain a sub-optimal solution. A disadvantage of the distributed optimization approach to CVEM is that the global optimality of the solution requires the components to be described by linear and (convex) quadratic models. This renders the distributed optimization approach not usable for the applications where non-linear models are required. However, particular extensions of the distributed optimization approach that include non-linear models have already been explored. For instance, in [33], a non-linear battery ageing model has been introduced to the CVEM problem, while in [34], the inertia of the vehicle is considered as an energy buffer and connected to CVEM (see Section V-C for more details). In both cases, the potential of the distributed optimization approach to manage non-linear models has showed promising results; in that sense, a possible extension to [13] is the generalization of the distributed optimization framework towards non-linear formulations.
C. Eco-driving

In the aforementioned approaches, the vehicle velocity (and thereby the power needed to propel the vehicle) is often assumed to be completely given a priori. Still, the vehicle inertia, which is the largest energy buffer in the vehicle can have a large impact in energy savings and consequently in the extension of the driving range. For instance, in [35] it has been reported that changes in driving behavior could improve the energetic performance of the vehicle more than 30%. These promising improvements in energy efficiency have contributed to the emergence of the eco-driving concept, which aims to increase the energy efficiency of a vehicle by means of a convenient selection of driving strategies; i.e. laws, technological implementations or simply changes in the drivers behavior. Hence, it is clear that eco-driving is a broad definition where government, manufacturers and users participate [36]. The problem of optimizing the velocity profile (in isolation form the other subsystems in the vehicle) to achieve optimal energy consumption has been considered for conventional vehicles in [37]-[39], for hybrid electric vehicles by [40], [41], and for electric vehicles by [42]-[46].

To solve the eco-driving problem, standard techniques used in optimal control have been adopted. In [47], [48] dynamic programming (DP) has been used. Alternatively, Pontryagin’s minimum principle (PMP) has been used in [43], [49], [50] and [42] to solve the optimal control problems presented. The main disadvantage is that PMP only provides a necessary condition for optimality and cannot incorporate state constraints easily. Therefore, in [39], [40], [46] static non-linear optimization techniques are used to solve the problem in presence of state constraints. In order to guarantee that these static optimization techniques and the methods based on PMP find global optimal solutions, it is important to understand convexity of the eco-driving optimal control problem. Unfortunately, the literature related to this topic is scarce. The noticeable exception is [40], where the continuous-time optimal control problem is approximated to a convex formulation. However, the possible loss of convexity in the discretisation step (to arrive at a finite dimensional optimization problem) has not been considered in [40]. As consequence of this limitation, in [51], conditions for convexity of the eco-driving control problem are presented (see Section V-B).

Eco-driving solutions have been widely implemented in Eco-Driving Assistance Systems (EDAS). Depending on the method used to influence the driving profile, the implementation of eco-driving solutions can be seen as an advisory system, where the driver receives suggestion to adjust the driving style to save energy consumption [52] or as adaptive cruise control (ACC) system, where the vehicle takes control on the velocity profile [53].

D. Preview Information

The energy performance for real time implementations of classic EMS and CVEM strategies strongly depends on the quality of the predictions that are used to solve the optimal control problem. In the case of eco-driving, complete information related to the driving profile is not necessary; however, this approach assume no obstacles in the path, e.g., traffic lights, intersections, vehicles, etc. This implies that the performance of the real time eco-driving solutions can be improved using accurate predictions of the the environmental conditions that surrounds the vehicle. Therefore, several methods to improve the preview information used in real time vehicle energy management applications have been reported in literature.

In [39], traffic information is integrated into the energy management problem as time and space varying velocity constraints that are updated every five minutes by the Freeway Performance Measurement System [54]. Similarly, in [55], the eco-driving problem is solved considering traffic conditions; the authors propose a method to automatically create a velocity corridor from statistical real truck operation data that is used to represent varying traffic flow. For both of the previous approaches, the results presented are promising; however, these methods are mainly applicable in highways where the influence of vehicles, traffic lights of intersections are not significant for the overall performance.

In the last years, energy management in urban traffic environments has become a relevant research branch. For this case, the preview information is mainly obtained by predictions based on data from sensors and communication networks, e.g., vehicle to vehicle (V2V) or vehicle to infrastructure (V2I). Approaches to solve the eco-driving problem for a road segment with several traffic lights have been reported in [56], [57]. In [58], [59] energy optimal adaptive cruise controllers are proposed. This approaches uses sensor information and communication V2V to design energy optimal velocity profiles constrained by the interaction with other vehicles in urban environments.

Interestingly, most of cases previously described only focus its attention to optimize velocity profile neglecting the influence of other subsystems in the energy management problem.

IV. Research Outlook

In this section, the main research question of this work is stated and the related implications to this question are discussed as problems and possible solution paths to follow.

The state of the art previously discussed shows that a large portion of the research conducted on energy management for automotive applications have been devoted to optimize the power split among the the subsystems in the vehicle, i.e., classic EMS and CVEM strategies; while the other large sector has been oriented to optimize the velocity profiles of the vehicle. However, the literature that links both of these large research fields is still scarce, albeit the significant improvements in performance and implementability that this research direction promises.

It is possible to consider the classic EMS as a subset of CVEM strategies. The main disadvantage in this case is the strong dependence on a priori information of the driving cycle, i.e., the velocity profile for a known trajectory in specific time interval. This driving cycle can be directly
obtained as a solution of the eco-driving problem, thus overcoming in some degree the main disadvantage present in CVEM strategies. As a consequence, the following research question is posed:

“What are the potential advantages of including the optimization of velocity trajectories in the CVEM framework for applications in electromobility?”

The implications that arise from the exploration of this question are discussed in the following Sections.

A. General framework for modeling and optimal problem formulation.

Problem Overview:

Obtaining optimal velocity profiles in the CVEM strategy directly implies that velocity becomes a decision variable in the energy management optimal control problem. The current modeling framework used in CVEM applications describes a network of subsystems where the iteration between them is based on energy flows. In that sense, the subsystems are modelled as energy buffers or energy converters and the formulation of the optimal control problem uses as decision variables the power produced / consumed by the subsystems. Therefore, understanding what is the most beneficial way to include velocity in the CVEM optimal control problem formulation becomes a priority task. For instance, [11] proposes a formulation where space is selected as the independent variable in order to avoid the non convexity introduced by the inclusion of velocity in the optimization problem; however, this produces non-linear and possibly non convex models for other subsystem that are normally linear and convex in formulations where time is considered as the independent variable.

The current modeling process tends to simplify the non-linearities in the models and mostly aims obtain to convex approximations that are convenient in view of the optimization methods performance. It is clear to note that these approximations introduce errors that can constraint the performance of the CVEM strategies. Therefore, it is relevant to investigate what are the problems and potential solutions that appear as a consequence of increasing the complexity of the models used for CVEM. An early exploration in this direction was already reported in [33], observing that the including ageing in the battery model changed the behavior of the CVEM strategy, i.e., increasing the complexity of the models to capture additional physical phenomena in the CVEM can lead to an improved performance of the strategy.

Research Paths:

In order to address these open topics on modeling, the first necessary step it to formalize and generalize a modelling framework for CVEM applications on electromobility. A generalized framework facilitates the design of systematic methods to obtain CVEM strategies, hence accelerating the study of the benefits for different formulations of the energy management problem. An early idea of the features expected in this modeling framework are already presented in Section V-A. In this case, non-linear models are considered and the states of the energy buffers are explicitly expressed in the network; furthermore, the use of time or space as the independent variable in the model is also envisioned. Thus allowing a the use of more detailed models to improve the performance of the CVEM strategy.

The implications of the level of accuracy required in the models is other topic that needs to be studied. Physically accurate models impose a higher computational effort to find the optimal solution of the energy management problem. The trade off between accuracy and complexity of the models is not a trivial task. One of the main issues is that the complexity of the models introduces non-convexity in the optimal control problem formulation. This problem can be addressed by the application of lossless convexification techniques. However, it is not always possible to successfully apply this approach; therefore, a detailed analysis of the convexity properties of the optimization problem is useful. This analysis helps to identify which physical properties can be relaxed or even removed from the formulation producing the minimum impact in the performance of the EMS. The idea is to systematically qualify and quantify the errors introduced in the formulation and use this insight to make decisions about the convexification techniques that can be applied to the CVEM problem. In [51], a first attempt in this research line has been presented. The convexity of the eco-driving problem in isolation was analyzed, thus identifying realistic assumptions and conditions that permit the formulation of a convex eco-driving optimization problem. These ideas can be extended to more complex formulations where additional non-convex subsystems are included in the CVEM problem.

B. Numerical Optimization Methods for CVEM

Problem Overview:

As it was already stated in the previous section, the use of richer models can lead to non linear and non convex formulations of the CVEM problem. This issue could be partially addressed by the use of convexification techniques in the formulation of the problem. However, for some cases those techniques fail to obtain formulations where convex optimization techniques can be used to efficiently solve the CVEM problem. This issue directly affects the real time implementation of CVEM strategies. In that sense, it is important to explore what optimization methods can be suitable to efficiently solve the CVEM problem under non linear and non convex formulations.

Research Paths:

Since the distributed optimization approach for CVEM studied in [13] has reported satisfactory results in terms of computational performance for off-line and on-line applications, it can be considered a promising method include the optimization of velocity profiles in the CVEM problem. However, the convergence in several of the algorithms proposed has not been guaranteed. In order to improve
the reliability of the distributed optimization approach it becomes relevant to explore convergence properties of the algorithms proposed in [13].

Additionally, in [13], alternating direction method of multipliers (ADMM) was used to split the horizon of the CVEM problem into small sub-problems that could be solved in a more efficient manner. The results reported still show a considerable computational effort to obtain the optimal solution of the problem. Thus, research should be conducted on this topic in order to adapt or obtain faster algorithms and also explore how these methods can exploit the structure of the problem in order to be used in non-convex formulations.

Algorithms that can tackle possibly non-convex formulations of the CVEM problem have been recently explored. For instance, sequential quadratic programming (SQP) and regularization techniques have presented promising results in [34]. This research line could be extended to achieve fast optimization algorithms that can be implemented in real time applications.

C. Preview Information using Sensor and Communication Networks

Problem Overview:
Considering velocity as a decision variable for the CVEM problem relaxes the requirement of a priori information related to the driving cycle. The additional freedom provided to the optimal control problem implies that a reduced set of information from the trajectory is needed. For instance, the eco-driving problem normally requires the position and velocities at the starting and final points of the trajectory and the complete road grade. Hence, it is clear to see that providing this information is less complicated than providing a detailed velocity profile, which in most of the cases is sub-optimal. However, this approach assumes ideal traffic conditions in the trajectory, e.g., a road without traffic lights, intersections and vehicles. Clearly, traffic conditions limits the performance of the energy management strategies, therefore it is relevant to investigate what preview information can be used to improve the performance of the CVEM strategies under traffic conditions and how this information can be used in the CVEM framework.

Research Paths:
It can be argued that for specific cases, e.g., a vehicle driving in a highway, the assumption of ideal traffic conditions holds. However, it is difficult to predict when this idealized case can be expected for an specific road. On the other hand, for urban traffic applications this idealized assumption is virtually impossible to satisfy.

In Section III-D, it was reported that several attempts have already been made to obtain and use preview information related to the traffic conditions. A large majority of the current work has been oriented to design cruise controllers that react to the environment conditions; unfortunately, only a reduced amount of these approaches explicitly considers energy consumption in the problem. Nevertheless, some of the concepts can be adapted to be used in energy management applications.

The use of statistical information to predict a velocity corridor that simulates traffic conditions for a specific trajectory was presented in [55]. In this case, the preview information is encoded as position dependent velocity boundaries that allows to reduce the energy consumed for braking. This approach can be extended to consider data form operation, but also climate conditions and time, which are parameters that influence the traffic flow.

Additionally, it can be noted that in the previous approach traffic lights or intersections are not considered. However, this information is available in cases where the vehicle uses communication networks I2V and I2I. This preview information can be used to obtain efficient management strategies when approaching to traffic lights or intersections. Moreover, the use of sensors in the vehicle give the possibility to define energy efficient cruise controllers that react to the behavior of the vehicle in front.

The applications previously mentioned are currently being explored by several authors; unfortunately, many of these approaches are disconnected from the energy management perspective. Including these ideas to the CVEM framework is not a trivial task. The main challenges are to investigate how the formulation of the CVEM problem has to be adapted and what optimization techniques can lead to the expected performance. Additionally, this implies that the CVEM concept can consider additional features, thus the list presented in Section II-A can be updated as:

- Globally Optimal.
- Scalable.
- Reconfigurable / Flexible.
- Anticipative.
- Reactive.

These new features are related to the way in which the preview information is used. An 'Anticipative' energy management strategy implies the exploitation of the direct information received from the communication networks and the predictions obtained form statistical models to react in advance to traffic conditions. The 'Reactive' property refers to the capacity to react to events that were not predicted or communicated in advance, e.g., vehicles that are driving in front without communication capabilities.

V. PRELIMINARY RESULTS

A. Power Network Modeling

The formulation of optimal energy control problems requires a proper description of the power flows among the subsystems that operate in the vehicle.

From the energetic perspective, the basic functions of a subsystem are to store and convert (and/or dissipate) energy between different physical domains [10]. For instance, a fuel cell uses the chemical energy stored as hydrogen to obtain electricity by means of a chemical reaction that occurs in the cell; similarly electric energy is transformed into mechanical energy by an electric machine. Hence, a subsystem $m \in \mathcal{M}$, is represented by an energy buffer connected to a power converter $c_m$, as can be observed in Fig.
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Fig. 3: Basic elements for modeling the vehicle power network.

3 [60]. The dynamical behavior of the buffer is represented by
\[
\begin{align*}
\dot{x}_m(t) &= f_m(x_m(t), u_m(t)), \\
z_m(t) &= g_m(x_m(t), u_m(t)),
\end{align*}
\]
where the mappings \( f_m : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n \), \( g_m : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R} \). The is the energy stored in the subsystem is \( z_m(t) \) and \( x_m(t), u_m(t) \) represents the states and the inputs of the buffer respectively. The energy conversion in \( e_m \) is given by
\[
y_m(t) = h_m(u_m(t)),
\]
where \( h_m : \mathbb{R}^n \to \mathbb{R} \), \( y_m(t) \) is the power of subsystem \( m \) flowing in the vehicle power network. Note that (3) describes the efficiency of subsystem \( m \) to convert energy. It is important to remark that it is also possible to have subsystems without energy buffers, for those cases \( h_m : \mathbb{R} \to \mathbb{R} \) defines a mapping between power in different energy domains.

The iteration of power flows can be observed in the nodes of the network, thus, for all the nodes \( j \in J \) it is possible to see that
\[
\sum_{P_j \subseteq P_f} y_{P_j} = 0
\]
where \( P_j \subseteq M \) is the subset of power flows interacting in the node \( j \). Note that by convection \( y_{P_j} \) is positive when it is reaching the node; for cases where the flow is bidirectional the positive flow direction can be selected arbitrarily.

B. Convexity of the Eco-driving Problem

This section summarizes the work presented in [51], where a detailed view of the convexity issues of the eco-driving optimal control problem is described and a method to reformulate and discretize the problem preserving its convexity is proposed. Furthermore, physically realistic assumptions and conditions that guarantee the convexity of the eco-driving problem are given and a Sequential Quadratic Programming algorithm that exploits these results is proposed.

1) Optimal Eco-driving Control Problem:

Eco-driving aims at obtaining an optimal traction force \( u(t) \) and velocity profile \( v(t) \) that minimizes the total power \( P(v, u) \) consumed by a vehicle while traveling during a given time interval \([t_0, t_f]\) over a given trajectory \( s(t) \in [s_0, s_f] \) with known geometrical characteristics, i.e., with a given road grade \( \alpha \in [s_0, s_f] \to [-\frac{\pi}{2}, \frac{\pi}{2}] \), where \( \alpha(s) \) is the grade at position \( s \); while being subject to longitudinal vehicle dynamics, non-negative velocity bounds \( v(t) \in [\bar{v}, \bar{v}] \), and boundary conditions on position and velocity. This can be stated in the form of the following optimal control problem:

\[
\begin{align*}
\min_{u(t),v(t),u(t)} & \int_{t_0}^{t_f} P(v(t), u(t)) \, dt \\
\text{subject to} & \quad u = m \frac{d^2 v}{d t^2} + mg(\gamma_r(s) + \gamma_f(s)), \\
& \begin{align*}
\frac{d^2 v}{dt^2} &= v, \\
v(t_0) &= v_0, \\
v(t_f) &= v_f
\end{align*}, \\
& \begin{align*}
\gamma_r(s) &= \sin(\alpha(s)), \\
\gamma_f(s) &= \cos(\alpha(s)).
\end{align*}
\end{align*}
\]

(5a)

In (5b), \( m \) represents the combined mass of the vehicle and the inertia of the driveline, \( g \) is the gravitational constant, \( c_r > 0 \) describes the rolling force coefficient and \( \sigma_d = \frac{1}{2} \rho_c \alpha A_f \) with \( \rho_c > 0 \) is the drag coefficient, in which \( \rho_c \) denotes the air density and \( A_f \) is the frontal area of the vehicle.

The consumed power \( P(v, u) \) can be obtained from different modeling approaches that capture the energy consumption in the powertrain. In this paper, it is assumed to be a quadratic function of the form
\[
P(v(t), u(t)) = \beta_0 v^2 + \beta_1 v + \beta_2 u^2 + \beta_3 v^2 + \beta_4 v + \beta_5 u^2 + \beta_6 v + \beta_7 u + \beta_8 v + \beta_9 u,
\]
for some non-negative parameters \( \beta_0, \beta_1, \beta_2 \). Equation (7) is a physically realistic approximation, e.g., for electric motors due to the fact that the friction and Ohmic losses are captured by the terms \( \beta_3 v^2 + \beta_4 v + \beta_5 u^2 + \beta_6 v + \beta_7 u \), respectively.

In general, (5) is a non-linear optimal control problem that might be nonconvex due to specific features of the vehicle model and road profile, see (5b). Furthermore, it has been proved that a direct discretization of the eco-driving can lead to a nonconvex optimization problem even for specific cases where the continuous time problem is convex. This implies that direct optimization approaches and methods based on PMP only provide candidate minima, which might not correspond to the global solution to problem (5).

2) Assumptions and Conditions for Convexity:

It is convenient to obtain an equivalent optimization problem that can be discretized using a forward Euler method while preserving its convexity properties. This goal is achieved by the substitution of (5b) into (5a) to obtain

\[
\begin{align*}
\min_{\alpha(t),u(t),v(t)} & \int_{t_0}^{t_f} P_R(a, s, v) \, dt, \\
\text{subject to} & \begin{align*}
\frac{d^2 v}{dt^2} &= v, \\
v(t_0) &= v_0, \\
v(t_f) &= v_f
\end{align*}, \\
& \begin{align*}
\gamma_r(s) &= \sin(\alpha(s)), \\
\gamma_f(s) &= \cos(\alpha(s)).
\end{align*}
\end{align*}
\]

(8a)

and
\[
\frac{d^2 v}{dt^2} = a; 
\]

(8b)

where \( a(t) \) is a new decision variable, which represents the vehicle acceleration. In this case the cost function is defined by

\[
P_R(a, s, v) = \beta_0 v^2 + \beta_1 v(\sigma_d v^2 + c_r m \gamma_r(s)) + \beta_2 (ma)^2 + \beta_3 (m \gamma_f(s) + \gamma_f(s))^2 + 2\beta_4 m^2 g v u (\gamma_f(s) + c_r \gamma_r(s)).
\]

(9)
The reduced eco-driving problem (8) is convex under a condition imposed on the lower velocity bound. To present this result, we need the following assumption.

**Assumption 1:** The given road grade \( \alpha(s) \) and the corresponding elevation profile \( h(s) \) satisfies

\[
\frac{d^2 h}{ds^2}(s) = 0 \quad \text{and} \quad \frac{d^2 \alpha}{ds^2}(s) = 0.
\]

where

\[
\gamma_d(s) = \frac{u}{v}.
\]

Under this assumption, a sufficient condition for global optimality of (8) can be derived, which are presented in the following theorem.

**Theorem 1:** Suppose that the optimization problem (8) satisfies Assumption 1. Then, the problem (8) is convex if the lower bound on the velocity \( v \) satisfies

\[
v \geq v_o(s) = \min_v \left\{ v \mid H_{33}(s, v) \geq 0 \right\}
\]

for all \( s \in [s_1, s_f] \), where

\[
H_{33}(s, v) = 2\beta_0 + 6\beta_1 \sigma_o v + 12\beta_2 \sigma_o^2 v^2 + 4\beta_2 \sigma_o^2 \mu \gamma_d(s) + c_r \gamma_d(s).
\]

The result from Theorem 1 imposes \( v_o(s) \) as the lower bound on the velocity that the vehicle has while driving downhill to guarantee a convex formulation of the optimization problem (8). This condition is satisfied for the majority of practical cases. For instance, in Fig. 4a, it is possible to observe the minimum velocity imposed by this condition for a vehicle with parameters presented in Table I. Recalling that \( v_o(s) = \sin(\alpha(s)) \), it can be observed that the vehicle could drive downhill with no velocity limitations for slopes larger than \(-20^\circ\). Even for roads with pronounced decreasing slopes, the minimum velocities allowed are still modest, which means that the condition stated in Theorem 1 is not highly restrictive for real-world applications.

Finally, it should be noted that Assumption 1 is an acceptable approximation. Indeed, for realistic roads, the rate of change of the elevation profile is relatively constant for small intervals. This can be observed in Figs. 4b and 4c, where the first and second derivatives of \( \gamma_d \) and \( \gamma_r \) are depicted for a road with slopes \( \alpha(s) \in [-60^\circ, 40^\circ] \), which is an extreme case. In both cases, the derivatives are virtually zero, meaning that Assumption 1 holds for a numerical solution to the optimal control problem (8). We will exploit this property in the numerical scheme that we will present below.

3) Numerical Example:

A Sequential Quadratic Programming (SQP) method that takes advantage of results from Theorem 1 has been proposed to solve the finite dimensional version of the eco-driving problem (8). Specifically, this formulation uses QP sub-problem that represents a convex second-order approximation of (8). The effectiveness of this methods is presented in a numerical example where a conventional driver behavior is compared with an eco-driving strategy for a heavy-duty vehicle. The settings used in this example are summarized in Table II and the elevation profile is defined by

\[
h(s) = 50 \cos\left(\frac{\pi}{20}\cdot s + \frac{\pi}{4}\right) + 50 [\text{m}],
\]

which is depicted in Fig. 6 as a green surface.

**TABLE II: Eco-driving problem parameters for a heavy-duty vehicle.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_0 )</td>
<td>0.292</td>
</tr>
<tr>
<td>( \beta_1 )</td>
<td>1.005</td>
</tr>
<tr>
<td>( \beta_2 )</td>
<td>2.652\times10^{-4}</td>
</tr>
<tr>
<td>( \sigma_o )</td>
<td>6.005 [\text{m/s}^2]</td>
</tr>
<tr>
<td>( \gamma_o )</td>
<td>0.1</td>
</tr>
<tr>
<td>( \gamma_d )</td>
<td>3.1246</td>
</tr>
<tr>
<td>( \gamma_d )</td>
<td>0.0055</td>
</tr>
</tbody>
</table>

The convexity of the optimization problem can be verified using (12). In Fig. 5, the satisfaction of this condition is depicted for all \( s \in [0, 21] \) [\text{km}]. Thus, form Theorem 1 it can be concluded that the problem in this example is convex.

The presumed conventional driver behavior is described by a solid line in Fig. 6. In this case, the total energy consumed by the vehicle is \( 6.218 \times 10^4 [\text{kJ}] \). On the other hand, the dashed line presented in Fig. 6 describes the velocity profile obtained as a solution of the eco-driving optimal control problem studied in this case. For this strategy the acceleration and deceleration stages are performed at reduced time intervals with larger magnitudes. Consequently, the average velocity achieved during the trip is considerably lower than the average velocity for the conventional case. The total energy consumed by the vehicle under this strategy is \( 5.165 \times 10^4 [\text{kJ}] \), which is approximately 16.92% lower than the energy consumed by the vehicle under conventional driving behavior.

Recalling that the optimization problem was certified to be convex a priori, it can be stated that the reported solution is an accurate approximation of the global solution of the eco-driving problem analysed in this example.

C. Complete Vehicle Energy Management and Eco-driving

The main result presented in (34) is briefly summarized in this section. Here, a first attempt to connect CVEM with eco-driving is proposed. Based on the distributed optimization approach of [61], the optimal velocity profile and power split of energy is obtained. The optimization sub-problem related to eco-driving is solved by the application of Thikonov regularization to an SQP formulation.

1) Problem Formulation:
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Electric Vehicle Enhanced Range, Lifetime And Safety Through INGenious battery management  

![Graph](image1.png)  
(a) Minimum downhill velocity required to guarantee convexity of the problem.

![Graph](image2.png)  
(b) Derivatives of $\gamma_g(s)$ with respect to the displacement $s$. 

![Graph](image3.png)  
(c) Derivatives of $\gamma_f(s)$ with respect to the displacement $s$.

Fig. 3: Example illustrating Assumption 1 and Theorem 1.

![Graph](image4.png)  
**Fig. 4:** Conventional vs. eco-driving profiles.

In CVEM, the main goal is to minimize fuel consumption, given by

$$
\sum_{k \in K} \gamma_{egu,k}, \quad (15a)
$$

subject to the dynamics and input-output behaviour of the converters in Fig 7, and the power balance at interconnection of the subsystems, i.e.,

$$
y_{em,k} - y_{egu,k} - y_{hvb,k} = y_{br,k} \leq 0, \quad (15b)
$$

for all $k \in K$. We use the constraint (15b) and energy balance constraint of the HVB, i.e.,

$$
\sum_{k \in K} \gamma_{hvb,k} = x_{hvb,K} - x_{hvb,0} = 0 \quad (15c)
$$

to rewrite (15a) as a ‘sum of losses’, i.e.,

$$
\tau \sum_{k \in K} y_{egu,k} - y_{egu,k} + y_{hvb,k} - y_{hvb,k} + y_{em,k} - y_{br,k} \quad (15d)
$$

By substituting (15b) and (15c) into (15d), we retrieve the original objective function (15a). The goal is to solve the CVEM problem by formulating it as a convex SQP problem and then apply dual decomposition as presented in [61].

2) Numerical Example:

In this simulation study, we consider the Series-Hybrid Electric Vehicle (SEHV) case study presented in V-C.1. We will refer to the ‘forward’ optimization as solving the vehicle energy management problem with eco-driving, and refer to the ‘backward’ optimization as solving an energy management problem without eco-driving, where the vehicle trajectory information, i.e., speed $v_k$ and distance $s_k$ for all $k \in K$, are given.

We base our case study on the work done in [11], in which a convex optimization approach is taken to solve the SEHV case study, where it is formulated a second-order cone program. Due to the chosen problem formulation, the authors in [11] have opted for a piece-wise linear EM model, linear EGU model and a quadratic HVB model. Furthermore, the authors in [11] define the optimization problem in the space domain, for which the physical interpretation of some parts of their problem formulation is not easy to understand. As we have defined the SEHV case study in the time domain, we do not face this issue. The parameters used for the simulation study are shown in Table III. We remark here that the upper and lower bounds for the state and input variables specified in Table III are defined for all $k \in K$.

The simulations are done for 1080 s over a distance of 21 km with a step size $\tau = 5$ s, which gives an optimization horizon $K = 216$. In ‘backward’ optimization, the speed is
given by a constant speed of \( v_k = 70 \text{ km/h} \) for all \( k \in K \), and in ‘forward’ optimization the initial and final velocity are given, while over the trajectory the speed is allowed to vary by \( 70 \pm 10 \text{ km/h} \). As initial conditions for the ‘forward’ optimization, we choose the solutions of the ‘backward’ optimization.

In Fig. 8, the simulation results for the ‘backward’ and ‘forward’ optimization method are given. In the ‘forward’ optimization results, we see that between 0 and 2 km, where the slope becomes negative, the vehicle reaches the lower speed bound. This action allows the available potential energy from the road profile, between 2 and 13 km, to be maximally converted to kinetic energy; the vehicle speed is maximized in this interval. After 13 km, the road gradient becomes positive and speed is minimized such that the final speed constraint is met. Therefore, we can see that as a result of having the speed as a decision variable, in the ‘forward’ case, the EGU may provide less power over the course of the trajectory, and noticeably less braking power is applied, when it is compared to the ‘backward’ case. The fuel consumption of the ‘backward’ and ‘forward’ simulation cases are 23.41 l/100 km and 22.31 l/100 km respectively. Thus, by including the eco-driving problem into the vehicle energy management problem, approximately 4.7% decrease in fuel consumption is achieved. This is comparable to the fuel consumption obtained in [11], which are 24.35 l/100 km and 23.98 l/100 km for the ‘backward’ and ‘forward’ case respectively. This is a 1.54% decrease in fuel consumption between the ‘backward’ and ‘forward’ case. We may explain this difference in fuel consumption savings largely due to the different models used.

\[ s_K = 21000 \text{ [m]} \]
\[ x_{v,b,p} = 11988 \text{ [kJ]} \]
\[ a_{v,b} = 2.22 \text{ [m/s]} \]
\[ v_k = 16.67 \text{ [m/s]} \]
\[ x_{hvb,b} = 22680 \text{ [kJ]} \]
\[ x_{v,b} = 22680 \text{ [kJ]} \]
\[ x_{v,b} = 2456 \text{ [kJ]} \]
\[ x_{v,b} = 92.4 \text{ [kW]} \]
\[ x_{v,b} = 92.4 \text{ [kW]} \]

In Fig. 8, the simulation results for the ‘backward’ and ‘forward’ optimization method are given. In the ‘forward’ optimization results, we see that between 0 and 2 km, where the slope becomes negative, the vehicle reaches the lower speed bound. This action allows the available potential energy from the road profile, between 2 and 13 km, to be maximally converted to kinetic energy; the vehicle speed is maximized in this interval. After 13 km, the road gradient becomes positive and speed is minimized such that the final speed constraint is met. Therefore, we can see that as a result of having the speed as a decision variable, in the ‘forward’ case, the EGU may provide less power over the course of the trajectory, and noticeably less braking power is applied, when it is compared to the ‘backward’ case. The fuel consumption of the ‘backward’ and ‘forward’ simulation cases are 23.41 l/100 km and 22.31 l/100 km respectively. Thus, by including the eco-driving problem into the vehicle energy management problem, approximately 4.7% decrease in fuel consumption is achieved. This is comparable to the fuel consumption obtained in [11], which are 24.35 l/100 km and 23.98 l/100 km for the ‘backward’ and ‘forward’ case respectively. This is a 1.54% decrease in fuel consumption between the ‘backward’ and ‘forward’ case. We may explain this difference in fuel consumption savings largely due to the different models used.

VI. FUTURE WORK

In this section, concrete plans for the immediate future research are presented. Specific details for the extensions of the preliminary results presented in Section V are given and a short discussion of a case study for CVEM on electric buses is summarized.

The next step in this research is to extend the results obtained in [51] to include force and power constraints in the convexity analysis. It has been observed that the same set of assumptions used in [51] can be used to obtain conditions for the convexity of the eco-driving problem under this new set of constraints. Additionally, guaranteeing global solutions in cases where the convexity condition (12) is satisfied only for a subset of the feasible is an interesting research extension. This could be achieved by proving the uniqueness of the critical point in optimization the problem, which seems feasible under the same assumptions presented in [51]. These additional results on the convexity and global solutions of the eco-driving problem can be connected to the work presented in [34]. Consequently, a convex formulation for the CVEM problem with velocity optimization could be proposed, thus also a reduced computational effort needed is expected. A case study to design CVEM strategies for electric buses is an research line that will be addressed. The main goal is to reduce the range anxiety of electric buses by improving the operational performance of the bus. There are two energy buffers that must be considered, i.e., the vehicle inertia and HVAC system. Bus manufactures have reported that the HVAC system normally is the first energy consumer, which is expected due to the large volume of the passenger cabin and the substantial exchange of energy with the external environment at every bus stop. Therefore, a CVEM strategy with velocity optimization that takes advantage of preview information of the next stop can be designed in order to exploit the capabilities of the energy buffers in the bus, hence maximizing the driving range of the vehicle. Additionally, a potential on-line implementation of the strategy is being considered, therefore the research activities will be aligned to this goal.

VII. CONCLUSIONS

In this paper, a view of the the future research on CVEM on electromobility has been presented. The implications of including velocity optimization in the CVEM framework have been discussed, thereby concluding that modeling, optimization methods and the exploitation of preview information are feasible research lines to follow. Consequently, ‘anticipation’ and ‘reaction’ were proposed as additional features of the complete vehicle energy management concept. Moreover, preliminary results that shows the potential of optimizing velocity profiles in the CVEM problem have been summarized. Finally, concrete plans for the extension of those results have been proposed and the design of a case study for electric buses was also discussed.

**TABLE III: Series Hybrid Electric Vehicle Parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m )</td>
<td>15050 [kg]</td>
</tr>
<tr>
<td>( g )</td>
<td>9.81 [m/s²]</td>
</tr>
<tr>
<td>( v_0 )</td>
<td>0.7</td>
</tr>
<tr>
<td>( \rho_0 )</td>
<td>0.0007</td>
</tr>
<tr>
<td>( A_f )</td>
<td>7.54 [m²]</td>
</tr>
<tr>
<td>( \mu )</td>
<td>1.184 kg/m²</td>
</tr>
<tr>
<td>( v_{in} )</td>
<td>19.44 [m/s]</td>
</tr>
<tr>
<td>( v_K )</td>
<td>( v_0 )</td>
</tr>
<tr>
<td>( \gamma_0 )</td>
<td>0 [m]</td>
</tr>
<tr>
<td>( s_K )</td>
<td>21000 [m]</td>
</tr>
<tr>
<td>( x_{v,b,p} )</td>
<td>11988 [kJ]</td>
</tr>
<tr>
<td>( a_{v,b} )</td>
<td>2.22 [m/s]</td>
</tr>
<tr>
<td>( v_k )</td>
<td>16.67 [m/s]</td>
</tr>
<tr>
<td>( x_{hvb,b} )</td>
<td>22680 [kJ]</td>
</tr>
<tr>
<td>( x_{v,b} )</td>
<td>7556 [kJ]</td>
</tr>
<tr>
<td>( x_{v,b} )</td>
<td>92.4 [kW]</td>
</tr>
<tr>
<td>( x_{v,b} )</td>
<td>92.4 [kW]</td>
</tr>
</tbody>
</table>

Fig. 8: Backward vs forward optimization. The dotted lines represent minimum and maximum state constraints.
REFERENCES


Range Estimation for Electric Vehicles

Francisco Ozuna

Abstract—Accurate range estimation in electric vehicles is a crucial challenge to increase customer trust in the use of batteries as an alternative source of power in mobility. The extractable energy in battery packs is determined by the amount of energy in the cells and the future energy consumption of the vehicle. Furthermore, a battery pack consists of hundreds of cells and having a state estimator for each cell is challenging for on-line implementation. Therefore, a low computational load algorithm is required to estimate the states of the pack and consequently the electric range. This paper presents a novel approach to estimate the range of an electric vehicle with a low computational load by estimating the battery pack state and forecasting the future power demand using available vehicle information. The results show that the driving range of an electric bus can be accurately estimated with a maximum range estimation error of less than 10%.

Keywords—Range estimation, Battery pack, Electric Vehicles.

I. INTRODUCTION

Electric vehicles (EVs) research is playing an important role in the development of alternative transport means to reduce the environmental impact of fossil fuels [1]. Nevertheless, there are some factors that discourage potential customers from buying EVs. Among these factors, driving range stands as one of the major issues. Most of current EVs have driving ranges around 400 km which is around 40% of similar sized internal combustion engine vehicles (ICEV) as can be seen in Fig. 1. Furthermore, the EV convenience usage is limited by the EV charging infrastructure. In the U.S. the number of public stations is estimated to be around 16,000 which is low compared to the estimated 112,000 gasoline stations. This means a reduced possibility to recharge an EV compared to an ICEV. Moreover, the gas refueling takes a couple of minutes to provide a range of approximately 300 miles for an average ICEV. Meanwhile, EVs require on average tens of minutes to hours to recharge [2].

For these reasons, people are still reluctant to acquire EVs due to the concern of how far they can travel before the battery depletes. For electric buses this is relevant as well. A service needs to be provided and it is important to know how far electric bus can travel and when they need to be recharged. The estimation of the distance which an EV can travel before the battery depletes, is henceforth referred to as range estimation.

The remaining range estimation of any electric vehicle can be computed using

\[ \text{Range}[\text{km}] = \frac{E^S[J]}{EV[J/km]} \]  

(1)

where \(E^S\) is the remaining available energy stored in the battery and \(E^F\) is the future energy consumption that the vehicle will require to complete a driving cycle.

There is much research on estimating the battery available energy. The research in [3],[4] over 350 sources of scientific literature where the methods related to this topic are reviewed. In [5], the remaining discharge (RD) energy in the battery pack is defined as the cumulative energy from the present condition to the state when one of the cells in the pack reaches the lower cut-off voltage and can be represented as

\[ E^{RD}(t) = \sum_{i=1}^{N} \int_{0}^{t} V^{oc,i}(\text{SoC})Q^i dt \]  

(2)

where \(E^{RD}(t)\) is the maximum RD energy at time \(t\), \(N\) is the number of cells in the battery pack connected in series, \(V^{oc,i}(\text{SoC})\) is the open-circuit-voltage (OCV) which is function of the state-of-charge (SoC) and \(Q^i\) is the maximum available charge of the \(i\)th cell. In this research, the remaining range is calculated considering the battery pack inconsistency and predefined driving cycles. The battery pack parameters are identified with a recursive least square (RLS) and the SoC is estimated using an unscented Kalman filter (UKF). As we can note, available energy in EVs is strongly dependent on the battery pack SoC. To estimate it, various approaches have been proposed [6]-[8]. In [9], Plett proposes the...
use of a method for pack state estimation called bar-delta filtering. An average SoC estimate at pack level is provided by an estimator called bar filter and the difference between each cell SoC and the average is given by an estimator called delta filter. The computational time required to estimate the battery pack SoC is reduced by executing the delta filter at lower rate than the bar filter.

There is some research focused on predicting $E^C$ in EVs [10], [11]. Wenjia Wang et al. [12] introduces a method to estimate how far an EV can travel by calculating the possible routes it can take from its actual location to the surroundings with a rough and precise estimation considering different factors that influence the future energy consumption, which is defined as:

$$E^C = f(E^{C\_\text{cruise}}, E^{C\_\text{RG}}, E^{C\_\text{acc}}, E^{C\_\text{AR}}, E^{C\_\text{ED}}, E^{C\_\text{DS}})$$

where $E^{C\_\text{cruise}}$ refers to the energy consumption due to vehicle speed, $E^{C\_\text{RG}}$ is the energy consumption related to road grade, $E^{C\_\text{acc}}$ is the energy consumption due to acceleration, $E^{C\_\text{AR}}$ is the aerodynamic resistance energy consumption, $E^{C\_\text{ED}}$ is the use of energy for electric devices and $E^{C\_\text{DS}}$ is the driving style which affects the energy consumption. These factors are determined by the use of a telematics center where the possible routes and vehicle status are calculated. Finally, range is computed using (1). However, this research considers the battery as a single cell; thus, ignoring potential imbalances within the pack and the battery efficiency losses are not considered. In [13], Liu et al. integrate the energy in the battery and future energy consumption to estimate the remaining discharge energy in a cell under certain dynamic operating profiles using predictive control theory and adaptive model prediction. This method consists of the use of a predetermined future current profile, estimation of battery present states and future variables such as SoC, voltage and battery parameters. This algorithm proves to be accurate to estimate the future available energy with an error below 2% of an EV for cell level with a given future current profile.

Although some results exist to estimate the remaining range for EVs, such methods do not consider jointly energy estimation in the battery and future energy consumption for battery packs. In [5], the available energy is estimated considering a battery pack with multiple cells with state estimation. However, the future energy consumption is not addressed in this research. In contrast, [12] predicts only the future energy consumption without considering battery state and energy losses due to current profiles. A research done in [13], estimates the available energy in a battery but at cell level with a predetermined future current profile. Battery packs typically consist of multiple cells connected in series and parallel. To estimate battery pack energy, a same number of algorithms would be required to estimate the battery pack state, which would represent a significant computational burden for the Battery Management System (BMS) requiring increased memory allocation.

In this paper, the remaining range of an electric vehicle is estimated using an approach with low computational complexity. Cells within battery packs may present imbalances; therefore, the state in the pack is estimated to obtain its energy. Meanwhile the forthcoming driving conditions are forecast to compute the future energy consumption by reading available on-line information from the EV. This research provides for the first time an integration of battery pack state estimation for available energy considering imbalances in the battery pack and prediction of energy consumption to estimate the remaining range of an EV. The energy in the battery pack is estimated by using the bar-delta filter principle from [9] and the future energy consumption is forecast by a low pass filter using past driving conditions. The remaining range accuracy is tested in an electric bus application and is computed using 3 algorithms to show improvement of range estimation with different energy approaches. The first approach calculates the remaining range without considering battery energy inefficiency, whereas the second approach considers the available energy in the battery via a forecast in current. Finally, the third approach estimates the battery energy via a predicted power demand.

This paper is organized as follows. Section II presents the SoC estimation for the battery pack to compute its energy. Section III describes the methodology to calculate the future energy consumption. Section IV introduces the 3 approaches to estimate the remaining range. Section V presents the implementation of the approaches and shows the results of range estimation for a model of an electric bus. Finally, Section VI contains the conclusions of this research.

II. BATTERY PACK STATE ESTIMATION

As presented in the introduction of this paper, SoC is an important state to estimate the available energy in the battery, and as consequence, relevant to compute the remaining range of any EV. Battery packs in EVs are comprised normally of hundreds of cells connected in series and parallel. These may present non-uniformities causing state and parameter differences. For this reason, it is important to track the state of each cell within the pack. However, having one estimator for each cell would represent a high computational burden for the Battery Management System (BMS). Therefore, this section introduces the process for estimating the state of each cell in the battery pack for multiple cells with low computational load for range estimation.

A. Cell Equivalent Circuit Modeling

To estimate the SoC, a cell model is needed. Since the low computational load implementation is important, a relatively simple Equivalent Circuit Model (ECM) is used
as shown in Fig. 2 [14], [15]. Then, the model is defined by
\[
\begin{align*}
\frac{d}{dt} \chi_{soc} &= \frac{1}{C_{batt}} I_{batt}(t) \\
\frac{d}{dt} V^{CI} &= -\frac{1}{R^{CI}} V^{CI} + \frac{1}{C^{CI}} I_{batt}(t) \\
\frac{d}{dt} V^{batt} &= V^{oc}(\chi_{soc}) + V^{CI} + R^{batt} (\chi_{soc}) I_{batt}(t),
\end{align*}
\]
where \(\chi_{soc}[\%]\) is the SoC, \(V^{oc}[V]\) is the open circuit voltage (OCV), \(I_{batt} [A]\) is the cell current, \(R^{CI}[\Omega]\) and \(C^{CI}[F]\) are the resistance and capacitance of the overpotential, \(R^{batt}[\Omega]\) is the cell’s series resistance and \(I_{batt} [A]\) is the cell current. Battery charging occurs when \(I_{batt} > 0\) and discharging when \(I_{batt} < 0\). The battery model parameters and OCV corresponding to a Li-ion cell are presented in Figures 3 and 4, respectively.

Li-ion batteries operate between well-defined voltage boundaries. During discharge, when the battery reaches a predefined voltage, the operation stops. These boundaries are defined as

\[V_{batt}^{\text{min}} < V_{soc} < V_{batt}^{\text{max}}\]

where \(V_{batt}^{\text{min}}\) is the minimum discharge voltage called cutoff voltage and \(V_{batt}^{\text{max}}\) is the maximum charge voltage.

By defining the battery states as \(x = [\chi_{soc} V^{CI}]^T\), (4) can be rewritten as the state space model
\[
\begin{align*}
\frac{d}{dt} x &= \begin{bmatrix} 0 & 0 \\ 0 & -\frac{1}{C^{CI}} \end{bmatrix} x + \begin{bmatrix} \frac{1}{R^{CI}} \\ 0 \end{bmatrix} I_{batt} \\
V_{batt} &= V^{oc}(1.0|x) + [0 1]x + R^{batt} I_{batt}.
\end{align*}
\]

**B. PACK SoC ESTIMATION WITH BAR-DELTA FILTER**

A typical approach for state estimation is to use an Extended Kalman Filter (EKF), which is a generalization of the Kalman Filter (KF) towards a non-linear system. To develop the SoC estimation based on the EKF, the system (4) is discretized at \(x_k = x(kT)\), \(k \in \mathbb{N}\) with sample time \(T > 0\), using a forward Euler discretization, leading to
\[
\begin{align*}
\dot{x}_{k+1} &= \begin{bmatrix} 0 & 0 \\ 0 & -\frac{1}{C^{CI}} \end{bmatrix} x_k + \begin{bmatrix} \frac{1}{R^{CI}} \\ 0 \end{bmatrix} I_{batt}^{k+1} \\
V_{batt}^{k+1} &= V^{oc}(1.0|x_k) + [0 1]x_k + R^{batt} I_{batt}^{k+1}.
\end{align*}
\]

The introduced discrete time ECM can be used to estimate the SoC in the battery. EKF uses first order approximations of the nonlinear dynamics in the error covariance propagation and consists of a prediction step
\[
\begin{align*}
\hat{x}_{k+1|k} &= A \hat{x}_{k|k} + B_k I_{batt}^{k+1} \\
\Phi_{k+1|k} &= A \Phi_{k|k} A^T + Q_k,
\end{align*}
\]
and correction step

\[
\begin{align*}
\hat{x}_{k+1|k+1} &= \hat{x}_{k+1|k} + K_k (V_{batt}^k - \hat{V}_{batt}^k) \\
\Phi_{k+1|k+1} &= \Phi_{k+1|k} - K_k \Phi_{k+1|k}
\end{align*}
\]  

(9)

in which

\[
K_k = \Phi_k C_k^T (C_k \Phi_k C_k^T + R_k)^{-1},
\]

(10)

and

\[
\hat{C}_k = \frac{\partial V_{batt}}{\partial x_k} |_{\hat{x}_k},
\]

(11)

where \(A\) and \(B\) represent the matrices from (7) and \(Q_k\) and \(R_k\) are the process and measurement noise covariance matrices, respectively.

Considering a high voltage EV battery pack, \(N\) cells are connected in series to supply the voltage demand. For instance, the trend in electric vehicles is towards battery pack voltages between 400-500 [V], which would require around 125 cells of 4 [V] connected in series. These cells may present non-uniformities causing state and parameter differences within the pack; therefore, all the cell states must be estimated. Assuming all cells are different and the difference is unknown, each cell model satisfies

\[
\begin{align*}
x_{k+1}^i &= A x_k^i + B I_{batt}^k \\
V_{batt, i}^k &= V_{oc}^i([1 0] x_k^i) + [0 1] x_k^i - R_{batt} I_{batt}^k
\end{align*}
\]

(12)

Nevertheless, having one estimator for each cell would represent a high computational burden to the Battery Management System (BMS). Instead of having \(N\) EKFs in parallel to estimate each cell state, the average SoC can be tracked by

\[
\tau_k = \frac{1}{N} \sum_{i=1}^{N} x_k^i,
\]

(13)

which leads to

\[
\begin{align*}
\tau_{k+1} &= \frac{1}{N} \sum_{i=1}^{N} x_{k+1}^i = A \tau_k + B I_{batt}
\\
V_{batt, i}^k &= \frac{1}{N} \sum_{i=1}^{N} V_{oc}^i([1 0] \tau_k) + [0 1] \tau_k - R_{batt} I_{batt}.
\end{align*}
\]  

(14)

The second equation in (14) holds approximately, because

\[
\frac{1}{N} \sum_{i=1}^{N} V_{oc}^i([1 0] x_k^i) \approx V_{oc}^i([1 0] \frac{1}{N} \sum_{i=1}^{N} x_k^i).
\]

(15)

and the smoothness of the OCV from Fig. 4. Furthermore, the differences between each cell and the average are tracked by

\[
x_{k}^{\Delta i} = x_{k}^i - \tau_k,
\]

(16)

which leads to

\[
\begin{align*}
\hat{x}_{k+1|k+1}^\Delta &= A \hat{x}_{k+1|k}^\Delta + K_k (V_{batt}^k - \hat{V}_{batt}^k) \\
\hat{C}_k &= \frac{\partial V_{batt}}{\partial x_k} |_{\hat{x}_k},
\end{align*}
\]

(17)

EKFs on (14) and (17) can be designed using (7). A bar filter estimates the average state in the pack and a delta filter estimates the individual state differences of every cell. To estimate the battery pack state, a bar filter and \(N\) delta filters are used. It would appear that the battery pack state estimation computational complexity has been increased from \(N\) to \(N + 1\), but this is not the case. An EKF is used to estimate the average state and a simpler EKF without coulomb counting and voltage drop calculation due to battery current is employed to estimate each state difference. Moreover, as the differences in the individual states change at a lower speed than the average state themselves, each cell \(\Delta\) state is estimated in a lower rate than the average pack state. In this case, the bar delta filter is estimated every 0.1 seconds and the delta filters are estimated every 100 seconds.

**C. Bar-delta filter performance**

In this subsection, the performance of the bar-delta filter is verified by first comparing the SoC of a modeled battery pack to an estimated SoC performed in a simulation environment with the bar-delta filter algorithm. Then the estimated SoC of a real battery pack is compared later to a true SoC of the same real battery pack obtained using coulomb counting with an accurate current sensor.

First, to obtain the SoC of a modeled battery pack, a driving cycle shown in Fig. 5 is simulated in an electric bus vehicle model. As shown in Fig. 6, the electric bus vehicle model, which is developed by TNO, simulates, among other subsystems, the powertrain of an electric bus. The powertrain is a forward facing model that transforms a driving cycle speed profile to an electric bus battery pack power demand. For practical data validation, the electric bus power demand is scaled to a power demand for a battery pack of \(N\) cells by

\[
P_{pack,N} = \frac{P_{pack,EBus} N_{pack}}{N_{EBus}},
\]

(18)

where \(P_{pack,N}\) is the scaled power demand corresponding to a battery pack of \(N_{pack}\) number of cells, \(P_{pack,EBus}\) is the original power demand in the electric bus and \(N_{EBus}\) is the total amount of number of cells in a battery pack of the electric bus. A battery pack model with power as input, is simulated with \(P_{pack,N}\) to obtain current in the battery pack \(I_{batt}\), terminal voltage and the SoC of every cell. The battery pack model consists of \(N_{pack}\) cells connected in series with the ECM model as shown in Fig. 7.

With the input conditions shown in Table I, the electric bus model is simulated to obtain the pack current, the
SoC and the terminal voltage of every cell. The estimated SoC for each cell is obtained by simulating the bar-delta filter approach with the electric bus model. For visual clarity, Fig. 8 shows the comparison between the SoC of the model and the SoC estimation from 3 cells of the pack using the bar-delta filter. The 3 cells are referred as cell A, B and C and correspond to the highest and lowest SoC in the pack and one cell in-between, respectively. Fig. 9 shows the estimation for these cells. It can be seen that the bar-delta filter is able to track the difference in SoC for each cell with low estimation error for the entire driving cycle. Furthermore, when the simulation stops with the conditions from (5), the cell in the pack with the lowest SoC is at 4% and the highest is at 7.5%, indicating that not all energy in all cells could be extracted.

To obtain estimated SoC of every cell from a real battery pack, a current profile is run in a pack comprised of 12 Molicel IHR18650A cells connected in series. The bar-delta filter estimates the state of each cell during the entire current cycle profile. To validate the algorithm, a laboratory test is performed to determine a posteriori the true SoC of every cell in the pack.

A Battery Test Setup is used to apply the current profile to the real battery pack. A diagram of this setup is shown in Fig. 10. The setup consists of 6 main components: a power supply (PS), electronic load (EL) and relay board which provide a real current based on current profile. A data-acquisition device reads CAN data from the battery pack BMS to provide measured terminal voltages of every cell and measured pack current to a PC running a Simulink® model which controls the tests and runs the bar-delta (BD) filter algorithm on-line to obtain the estimated SoC.

The battery pack contains 11 cells with an initial 90% SoC and one cell at 78% SoC, which is a realistic scenario if a pack is charged only with fast charging at constant current or a cell is substantially aged. The battery pack is discharged using the current profile without cell passive.
balancing. This allows to observe the voltage differences in the cells due to non-uniformity in the battery pack. For visual clarity, Fig. 11 shows the bar-delta filter estimation comparison of the average true SoC and the SoC of 2 cells in the pack. These 2 cells correspond to the highest and lowest SoC estimated by the delta filter and are referred as cells A and B, respectively. Fig. 12 shows the estimation error. It can be seen that the bar-delta filter is capable to track the SoC of a real battery with higher error than the simulated pack; however, the accuracy decreases due to impreciseness from the current and voltage measurement. Nevertheless, the SoC estimation is considered accurate since the maximum error is less than 2%.

Finally, the speed performance of the bar-delta filter is tested. The purpose of using this filter is to have an accurate and fast algorithm that estimates the state of each cell within the battery pack and therefore, a low computational load range estimation algorithm. Table II shows the computational load comparison between different bar-delta filter configurations for a battery pack of 12 and 96 cells. In all cases, the algorithm was simulated in Simulink\textsuperscript{R} with voltage and current profiles from a full driving cycle. The time to run all the algorithm is measured using a computer running Matlab\textsuperscript{R} function cputime with an Intel\textsuperscript{R} Core i7 @ 2.40GHz processor and 7.7 GB of RAM. It can be seen from this table that the more cells the battery pack contains, the more benefit of computational load is obtained using the bar-delta filter algorithm.

The tests presented in this section show that the bar-delta filter algorithm is capable of tracking on-line state...
variations in the pack for a real electric bus driving cycle with low computational load, which is relevant for the range estimation in EVs. Estimating the lowest SoC in the pack is important to estimate the $E^3$ in any EV as it determines the maximum extractable energy in the pack.

### III. FUTURE ENERGY PREDICTION

To predict the future energy consumption, the forecast of driving conditions is an important factor. This section introduces the methodology to predict the future cycle conditions of vehicle speed, pack current and power in the battery pack of an EV. The driving cycle forecast is a challenging task to solve since the driver can modify the driving behavior at any time and the initial conditions of any driving cycle will not define the total cycle output. However, past information of the drive cycle may provide data about future behavior.

#### A. Low pass filter and cut-off frequency selection

To forecast the driving conditions, the low frequencies of vehicle speed, pack current and power in the battery pack are projected into the future using a low-pass filter in real-time. The filters are given by

\[
\begin{align*}
\hat{v}_{veh}^{k+1} &= (1 - \delta f^*)\hat{v}_{veh}^k + (\delta f^*)v_{veh}^k \\
\hat{I}_{pack}^{k+1} &= (1 - \delta f^*)\hat{I}_{pack}^k + (\delta f^*)I_{pack}^k \\
\hat{P}_{pack}^{k+1} &= (1 - \delta f^*)\hat{P}_{pack}^k + (\delta f^*)P_{pack}^k,
\end{align*}
\]  

(19)

where $\hat{v}_{veh}^{k+1}$ is the vehicle speed prediction, $\hat{I}_{pack}^{k+1}$ and $\hat{P}_{pack}^{k+1}$ are the battery pack current and power prediction, respectively. A crucial decision when selecting this method is defining the low cut-off frequency $f^*$. To select this cut-off frequency, the Power Spectral Density (PSD) of the speed from different driving cycles need to be analyzed to check which frequencies have the most information about a driving cycle.

Fig. 13 shows the PSD of different driving cycles that last 1.5 hours each, including the electric bus speed profile from Fig. 5 and the standardized USA CITY II and USA FTP driving cycles. A complete driving cycle from the electric bus corresponding from full to empty battery is called electric bus complete, which lasts 6 hours. The speed profile from each driving cycle is shown as function of frequency and is normalized from 0 to 1 as

\[
v_{veh}^k(f)^* = \frac{v_{veh}^k(f)}{v_{veh}^k(f)_{max}}. 
\]  

(20)

The electric bus driving cycle contains 2 main sub-cycles of approximately 1.5 hours each, which represent short trips the bus takes from the bus station to a final destination. As the low bounds of the PSD are given by the signal length, most of the cycle information for electric bus cycles 1 and 2 is contained within the frequencies above $1.8 \cdot 10^{-4}$ Hz. Standardized USA CITY II and USA FTP 75 driving cycles are repeated to complete a 1.5 hours speed profile to emulate the driving profile of a bus inside a city. Since these cycles have the same cycle duration of 1.5 hours, the cycle information is also contained within the frequencies above $1.8 \cdot 10^{-4}$ Hz. Meanwhile, the complete electric bus driving cycle shows to contain most of the information above $10^{-5}$ Hz. This means that the cut-off frequency represents the time frame of past information that is filtered by (19).

In EVs, the available energy in the battery changes during the driving cycle; therefore, to predict the driving conditions, the span of time which is looked into the past has to be selected according to the level of energy available in the battery. For the previous reason, to predict...
the future driving conditions, the cutoff frequency of the first order low pass filter is set as

$$f^c = \begin{cases} 
4.5 \cdot 10^{-6} \text{ for } 75\% < \text{SoC} \leq 100\% \\
2.5 \cdot 10^{-4} \text{ for } 30\% < \text{SoC} \leq 75\% \\
1.5 \cdot 10^{-3} \text{ for } 0\% \leq \text{SoC} \leq 30\%
\end{cases}$$

(21)

where $f^c$ is the cut-off frequency of the low pass filter expressed in [Hz]. For high SoC values, the cut-off frequency considers a time frame of 6 hours of past information to predict the future. For SoC values between 30\% and 75\%, the time frame corresponds to 1.1 hours. Finally, for SoC below 30\%, the last 11 minutes of vehicle operation are used for prediction.

To predict future driving conditions, the filtered data of the speed, current and power is projected to the future using (19) and cut-off frequency conditions from (21) at each time step.

B. Energy Prediction performance

To measure the performance of the forecast algorithm, a real average is calculated a posteriori by

$$\hat{r}^{\text{veh}}_k = \frac{1}{M} \sum_{i=k}^{M} r^{\text{veh}}_i$$

$$\hat{r}^{\text{pack}}_k = \frac{1}{M} \sum_{i=k}^{M} r^{\text{pack}}_i$$

(22)

where M is the number of points from time step k until the end of cycle.

Fig. 14 shows the comparison between speed, current, power prediction obtained from predicting the future average signals using past information using (19) and the real averages calculated from (22) during the entire driving cycle from full to empty battery. It can be seen that the prediction is overestimated and underestimated for all driving conditions during different time steps.

For instance, when speed is over estimated in the first hour of the driving cycle, current and power are also over estimated. Furthermore, at the beginning of the driving cycle, the prediction has a smooth pattern since the driving conditions are being filtered with the lowest cut-off frequency. This means that the time frame of past information used for prediction corresponds to a full cycle of 6 hours. As the driving cycle ends, the time frame of past information used for prediction is reduced to 10 minutes using the cut-off frequency of $1.5 \cdot 10^{-3}$ Hz.

The low pass filter cut-off frequency increases to look for past driving conditions from a short period of time in the past.

has a delayed following pattern of the real future average which

becomes more sensitive when the battery gets closer to empty due to the conditions from (21).

Table III shows the maximum and average errors for speed, current and power for the entire driving cycle with a battery pack fully charged until cut-off voltage is reached.

**TABLE III. FORECAST ERROR FOR DIFFERENT TIME STEPS IN THE DRIVING CYCLE**

<table>
<thead>
<tr>
<th>Driving condition</th>
<th>Maximum Error</th>
<th>Average error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r^{\text{veh}}_k$</td>
<td>65.55%</td>
<td>18.57%</td>
</tr>
<tr>
<td>$r^{\text{pack}}_k$</td>
<td>84.17%</td>
<td>20.30%</td>
</tr>
</tbody>
</table>

This section introduced the forecast of energy consumption using a first order low pass filter, which provides a low computational effort algorithm. The filtered
values of speed, pack current and power are projected in the future to predict future driving conditions at each time step. The results reflect that the filter is able to predict driving conditions with an average error below 24%.

IV. RANGE ESTIMATION

This section introduces the estimation the remaining range of an electric vehicle employing the SoC estimated by the bar-delta filter from Section II and the prediction of vehicle driving conditions using the low pass filter from Section III.

From (2), range estimation can be expressed as

$$ R = \frac{E_{k}^{S}}{V_{k}^{oc}} = \frac{Q_{k}^{batt}V_{k}^{batt}}{I_{k}^{batt}V_{k}^{batt}} $$

(23)

where $E_{k}^{S}$ [J] is the energy estimation in the source, $E_{k}^{C}$ [J/km] is the future energy consumption, $I_{k}^{empty}$ [km/h] is a predicted speed and $V_{k}^{empty}$ [h] is the time the battery terminal voltage will take to reach cut-off voltage, which can be expressed as

$$ V_{k}^{empty} = \frac{E_{k}^{S}}{P_{k}^{batt}} = \frac{Q_{k}^{batt}V_{k}^{batt}}{I_{k}^{batt}V_{k}^{batt}} $$

(24)

where $P_{k}^{batt}$ and $I_{k}^{batt}$ are predicted power and current in the battery expressed in [J/h] and [A], respectively. $Q_{k}^{batt}$ is the estimated extractable charge in the battery expressed in [Ah] and $V_{k}^{batt}$ is the predicted terminal voltage in the battery.

The extractable charge in a battery is directly related to the energy which can be extracted from the battery. From the battery model in (7), when current is applied to the battery, power losses occur in $R_{0}$ and $R_{1}$ causing battery voltage to stop at cut-off voltage even if energy is still available in the battery, making the extractable energy $(E_{S,ext})$ less than the total energy $(E_{S,total})$ in the battery, hence, providing less extractable charge. Fig. 15 shows a representation of this energy difference, where the extractable charge is noted as $Q_{batt,ext}$ $(P_{k}^{batt})$ and $V_{soc,ext}$ is the open circuit voltage when cut-off voltage is reached.

The battery model in (7), consists of current as input and voltage as output. However, automotive applications require a constant power demand which needs to be delivered by the battery. Power in a battery can be expressed as

$$ P_{k}^{batt} = I_{k}^{batt}V_{k}^{batt} $$

(25)

By substituting (7) in (25), $P_{k}^{batt}$ can be described in terms of battery parameters as

$$ P_{k}^{batt} = I_{k}^{batt}(V_{soc} + V_{k}^{oc} + R_{1}I_{k}^{batt}) $$

(26)

The difference in battery behavior is evaluated by applying a constant current and constant power to the battery model. A simulation is performed with constant current using (7) and another simulation is run with constant power using (26). This comparison is shown in Fig. 16. It can be clearly seen that power decreases with a constant current demand profile, which is not representative for automotive applications. Furthermore, in order to keep a constant power demand, current needs to increase as battery voltage becomes lower, providing less extractable charge since the power losses are bigger with a higher current.

As it can be seen, any current demand will make the battery to deliver less energy than the total energy stored in the battery. Furthermore, the battery behaves differently with a power demand. In this view, the re-
mainining range estimation accuracy is benchmarked with 3 approaches:

1) Charge Based (CB) approach. All energy in the battery is considered extractable; therefore it does not depend on power or current prediction. The time to reach cut-off voltage is calculated by using (24) considering a constant battery voltage.

2) Voltage Based (VB) approach. It considers that extractable energy is less than the total battery energy. The extractable charge depends on a predicted current demand and the time to reach cut-off voltage is calculated using (24) with a constant battery voltage.

3) Power Based (PB) approach. It considers that extractable energy is less than the total battery energy. The extractable energy depends on a power prediction. The time to reach cut-off voltage is calculated using (24).

All approaches calculate the time that it will take the voltage of the cell with lowest SoC from present conditions to reach cut-off voltage with the steady state condition

$$\frac{d}{dt} V_{oc} = 0.$$  \hfill (27)

A. Charge based approach

The CB approach is a simple solution that estimates the remaining range without modeling the energy losses by considering that all charge is extractable from the cell with the lowest SoC.

The extractable charge in the battery is estimated by

$$\hat{Q}_k = \hat{\chi}_{soc,low} C_{batt}$$  \hfill (28)

where \(\hat{Q}_k\) is the estimated extractable charge in a battery and is expressed in [Ah], \(\hat{\chi}_{soc,low}\) is the lowest SoC from the pack estimated with the bar-delta filter. Then, the time that the battery will take to reach cut-off voltage is calculated with (24) using the current prediction in the battery. Considering a battery pack arrangement where all cells in parallel are equally balanced, current in the battery will be

$$I_k^{\text{pack}} \approx \frac{I_k^{\text{pack}}}{N_{\text{parallel}}}$$  \hfill (29)

where \(I_k^{\text{pack}}\) is the pack current prediction calculated from Section III and \(N_{\text{parallel}}\) is the number of cells in parallel in the pack. Finally, the remaining range is estimated with (23).

B. Voltage Based approach

The VB approach estimates the remaining range by considering the extractable charge which depends on a current prediction. The time to reach cut-off voltage is calculated from the cell with lowest SoC.

The extractable charge in a battery can be estimated by

$$\hat{Q}_k = (\hat{\chi}_{soc,low} - \hat{\chi}_{soc,cf}(I_k^{\text{pack}})) C_{batt}$$  \hfill (30)

where \(\hat{\chi}_{soc,cf}(I_k^{\text{pack}})\) is the SoC when cut-off voltage is reached in the same cell with lowest estimated SoC and is function of the current prediction. Considering the steady state condition from (27), the SoC at cut-off voltage can be calculated by

$$V_{oc,cf}^{\text{batt}}(\hat{\chi}_{soc,cf}) = \frac{V_{\text{batt}} - \int_{k+1}^{\text{batt}} (\hat{P}_k^{\text{batt}} + R_{k,cf}^{\text{batt}})}{\chi}$$  \hfill (31)

where \(V_{oc,cf}^{\text{batt}}\) is the OCV at cut-off voltage, \(R_{k,cf}^{\text{batt}}\) is the series resistance and \(R_{k,cf}^{\text{batt}}\) is the resistance of the over potential when cut-off voltage is reached. Using the OCV from (31), SoC at the cut-off voltage is obtained from the OCV-SoC relation shown in Fig. 4. Then, the time to reach cut-off voltage is calculated with (24). Finally, the remaining range is estimated using (23).

C. Power Based approach

The PB approach estimates the remaining range by considering an extractable energy as a function of the power prediction in the battery with lowest SoC in the pack.

The extractable energy in a battery as a function of power prediction can be calculated by

$$E_k^P = V_{\text{batt}}^{\text{batt}}(\hat{\chi}_{soc,low} - \hat{\chi}_{soc,cf}) C_{batt}$$  \hfill (32)

where \(E_k^P\) is the extractable energy in a battery and is expressed in [Wh], \(\hat{\chi}_{soc,cf}(P_k^{\text{batt}})\) is the SoC when cut-off voltage is reached in the same cell with lowest estimated SoC and depends on the power prediction. Considering the steady state condition (27), current in the battery can be written as

$$I_k^{\text{batt}} = \frac{V_{\text{batt}} - V_{oc}^{\text{batt}}(\hat{\chi}_{soc})}{R_0 + R_1^{\text{batt}}}.$$  \hfill (33)

OCV at cut-off voltage can be calculated by substituting (33) in (26) by

$$V_{oc,cf}^{\text{batt}}(\hat{\chi}_{soc,cf}) = \frac{\hat{P}_k^{\text{batt}} (R_0 + R_{1,cf})}{V_{\text{batt}}} + V_{\text{batt}}.$$  \hfill (34)

Considering that power prediction in a battery from the pack can be described as

$$\hat{P}_{k+1}^{\text{batt}} = \frac{\hat{P}_k^{\text{pack}}}{N_{\text{series}} N_{\text{parallel}}}.$$  \hfill (35)

where \(N_{\text{series}}\) and \(N_{\text{parallel}}\) are the number of cells in series and parallel in the pack, respectively.

To estimate the remaining range, first OCV at the cut-off voltage is calculated from (34), then SoC at the cut-off voltage is obtained through the OCV-SoC relation shown
in Fig. 4. Then, the extractable energy in the battery with lowest SoC is estimated with (32) and the time to reach cut-off voltage is calculated with (24). Finally, from (23), the remaining range is estimated.

V. RESULTS AND DISCUSSION

This section presents the remaining range estimation results from the approaches introduced in Section IV. As explained in Section II-C, a real driving cycle in 2 scenarios is simulated with an electric bus vehicle model to obtain speed, traveled distance, cell terminal voltage, cell SoC and battery pack current and power. The first scenario considers a balanced battery pack and the second one considers an unbalanced battery pack. The traveled distance in the vehicle model is the reference for the remaining range estimation in the 3 approaches: Charge Based, Voltage Based and Power Based. The remaining range estimation is compared to the reference traveled distance and the accuracy of the results are presented in both scenarios. Finally, a sensitivity analysis is performed to analyze and discuss the research results.

A. Remaining Range estimation results

The real range is obtained from simulating the electric bus vehicle model with the conditions presented in Table I in 2 scenarios:

1) Scenario 1. A balanced battery pack model of 12 cells is included in the electric bus vehicle model. All cells start at 92% SoC and have the same parameters shown in Fig. 3. The capacity of each cell is different and is assigned between 2.01 and 2.10 Ah.

2) Scenario 2. A battery pack model of 11 balanced cells and 1 unbalanced cell is used in the electric bus vehicle model. The 11 balanced cells start at 95% SoC with different capacities between 2.01 and 2.10 Ah and the unbalanced cell starts at 85% SoC with a capacity of 1.8 Ah. As described in Section II-C, having this unbalanced situation is a possible real scenario for battery pack which is fast charged with only constant current or with a substantially aged cell.

For each scenario, a repeated real driving cycle from Fig. 5 is simulated in the electric bus vehicle model until one of the cells in the battery pack reaches cut-off voltage. The SoC of each cell in the battery pack for scenario 1 and 2 are shown in Figures 17 and 18, respectively.

The electric bus speed, current and power in the pack are recorded from the vehicle model and then simulated with each range estimation algorithm. The range calculation is started in the moment the electric bus starts moving and the estimation algorithm is stopped while the electric bus is not moving. Figures 19 and 20 show the remaining range estimation results for each algorithm for scenarios 1 and 2, respectively.

For scenario 1, it can be clearly seen that Charge Based approach is overestimating the range for the entire driving cycle with an average error of 10.70 km and a maximum error of 20.38 km. On the other hand, Voltage and Power Based approach remaining range estimations are closer to the range reference with an average error of 3.02 km and 3.43 km, respectively, with errors below 10%. For scenario 2, the Charge Based approach has a bigger range overestimation with an average and maximum errors of 12.36 km and 22.74 km, however, Voltage and Power Based approaches keep a low range estimation error with average and maximum errors close to 4 km and 11 km, respectively.

A sensitivity analysis is done to analyze the effect of the prediction of the driving conditions in the range estimation for each algorithm. The range estimation is tested...
in 2 different scenarios of predicted driving conditions:
1) Predicted Average. Range estimation is calculated with predicted driving conditions from (19).
2) Real Average. A real predicted average of each driving condition from (22) is used as an input for each range estimation algorithm.

Furthermore, the influence of the bar-delta filter algorithm in the range estimation is tested by using the lowest or average SoC in the balanced and unbalanced packs. This SoC is used as an input for estimating extractable energy in (32) for the Power Based approach, the extractable charge in (28) and (30) for Charge Based and Voltage Based approaches, respectively. Finally, a battery model from (7) is used to benchmark the range estimation algorithms accuracy. The predicted and averaged currents are used to measure the time that it takes the cell with lowest SoC to reach cut-off voltage by simulating the battery model. The range is estimated using (23). Table IV shows the results of this sensitivity analysis including the testing of predicted and real average driving conditions.

As expected, by using the real average speed, current and power in the pack from (22), the range estimation becomes more accurate than using the predicted conditions form the low pass filter. Furthermore, Voltage and Power Based approaches show a similar accuracy while estimating the range. Both of them have similar maximum and average errors for a balanced pack and for an unbalanced pack.

Contrary to expectations, using the bar filter provides a slight better range estimation in a balanced battery pack for Voltage and Power Based approaches, where the average range accuracy is improved around 0.5 km. On the other hand, the bar filter worsens Charge Based range estimation accuracy. The estimation error for this approach is 10.70 km for the delta filter and 11.53 km for the bar filter.

Meanwhile, the range estimation in an unbalanced battery pack shows different results. The delta filter improves the range estimation in all the cases. The maximum and average errors using the delta filter are lower than the bar filter.

Using a battery model provides a range estimation error below 5 km for all average errors. The maximum errors are maintained under 10 km in all cases except when using the predicted average in an unbalanced pack. This indicates that using a battery model provides a range estimation close to the real range without high variation along the driving cycle.

B. Remaining Range estimation discussion

As it can be seen, the future energy prediction has a significant influence on the remaining range estimation. From Fig. 19, it can be noticed that the range estimation in the voltage and power approaches follow similar patterns as the results from the low pass filter shown in Fig. 14. When the prediction is overestimated, more speed, current and power are predicted. Therefore, the range is underestimated since the model forecasts that more energy will be consumed in the future, and ultimately, less remaining range is available. On the other hand, when the prediction is underestimated the opposite effect occurs.

Even though the forecast has a significant influence in range estimation, it is still important to consider
the energy that can be extracted from the battery. In the balanced and unbalanced packs, the Charge Based approach has a bad range estimation performance compared to the Voltage Based approach despite the fact that the same speed and current prediction were used for both algorithms. The Voltage Based approach had a better accuracy than the Charge Based algorithm since it estimates the extractable energy.

In the balanced pack, the delta filter improved the range estimation only for the Charge Based approach since this algorithm is initially overestimating a range. Therefore, the delta filter helps to lower the range estimation since less energy available is calculated. On the other hand, the bar filter makes the range estimation worse since it considers more extractable energy. With respect to the Voltage and Power approaches, using the forecast of driving conditions and estimating the extractable energy with the bar filter is enough to estimate the range with low estimation error. As a consequence, the delta filter worsens the range estimation as the future energy consumption is overestimated for most part of the cycle.

The opposite effect happens when using an unbalanced battery pack. The range estimation is improved by using the delta filter in all cases. The extractable energy in the pack is more accurately estimated than the bar filter since the lowest SoC is estimated. The dynamics of the cell with the lowest SoC determine the extractable energy in the pack, hence it is the limiting factor how far an EV can travel. In the case of the unbalanced pack, the difference between the lowest SoC and the SoC of the rest of the pack is significant; therefore, the delta filter estimates the extractable energy more accurately providing a better range estimation than the bar filter, which overestimates the extractable energy.

The purpose of benchmarking the accuracy of range estimation with the Voltage and Power approaches is to test the difference of using a current and power demand profiles as shown in Fig 16. The use of predicted current and power demand in the algorithms did not yield a significant difference in the range estimation for both, balanced and unbalanced battery packs. For instance, in the balanced pack, the Voltage Based approach had a better accuracy for 0.5 km in the average error throughout the entire driving cycle; however, the Power Based approach had less peak errors than the Voltage approach for less than 4 km.

### VI. CONCLUSIONS AND FUTURE WORK

This paper introduced the remaining range estimation for any EV based on the integration of states estimation in a battery pack using a bar-delta filter and the predicted driving conditions using a low pass filter with a low computational effort. The results show that range can be estimated with an error of less than 5% for a balanced pack and less than 7% for an unbalanced pack in a simulation study of an electric bus.

Contrary to expectations, the delta filter showed to have a low influence in range estimation for a balanced battery pack. Using the cell with the lowest SoC in the pack to calculate the time to reach cut-off voltage did not have a significant influence in the range estimation since the cell with the lowest SoC of the pack has not a considerable difference compared to the SoC of the rest of the cells. We assume that in the case of balanced packs, using a delta filter for this purpose is not as critical as forecasting accurately the driving conditions. However, results indicate that for an unbalanced battery pack, the delta filter provides a better range estimation than using the bar filter. We conclude that predicting the driving conditions has more influence in the range estimation when the battery pack is balanced. Nevertheless, as the battery pack ages, unbalances might become more significant. Furthermore, a fast charging session with only a constant current might lead to have unbalances in the pack as well. Therefore, the bar-delta filter algorithm takes more relevance as the battery ages and it should be considered to estimate the range of an EV.

Using past information to predict the future demonstrated to be helpful to estimate the remaining range. The predictions in this research had an average error of around 20%, which still provided range estimation results below 10% error. Having a precise prediction of future driving conditions is a challenging task. However, there can be alternatives to improve the accuracy of the forecast. The prediction of driving conditions can be improved by having an adaptive and self learning low pass filter according to present driving conditions. Furthermore, incorporating more information such as GPS route and traffic data might increase the accuracy of the forecast. In the case of electric bus applications, the energy consumption can be linked to the time of the day of the driving cycle. For certain times of the day, different energy consumptions can be predicted by using peak and off-peak hour information such as number of passengers
and traffic. The repetitiveness of the driving cycles for public transport can be an opportunity to conduct a further research on the use of filters to estimate future driving conditions.

VII. REFERENCES

REFERENCES

A Global Optimal Solution to the Eco-Driving Problem

G.P. Padilla, S. Weiland, M.C.F. Donkers

Abstract—Eco-driving aims at minimizing the energy consumption of a vehicle by adjusting the vehicle's velocity. This can be formulated as an optimal control problem and this paper provides a detailed view on the global optimal solution to this problem. A method to reformulate and discretize the problem avoiding the introduction of additional nonconvex terms is presented. Furthermore, physically realistic conditions are given that guarantee the existence of the global optimal solution to the eco-driving problem. Subsequently, a sequential quadratic programming algorithm is provided that allows finding the global optimal solution. Finally, two numerical examples are used to illustrate how solutions of the eco-driving problem can be obtained.

Index Terms—Automotive control, Optimal control, Optimization.

I. INTRODUCTION

Improving energy efficiency of vehicles is an important topic of research for the automotive industry. High energy efficiency is important for reducing fuel consumption and meeting emission legislations. Moreover, energy efficiency is also supported by the functional argument of mitigating range anxiety of electric vehicles, i.e., giving a sufficiently large driving range for an electrical vehicle. The problem of reducing the energy consumption of a vehicle over a certain drive cycle can be formulated as an optimal control problem and its solution is often referred to as an energy management strategy. Most of these energy management problems are focused on controlling the power split between the combustion engine and the electric machine of a hybrid electric vehicle [1], [2]. By storing regenerative braking energy and shifting the operating points of the combustion engine, a significant amount of fuel can be saved. A recent trend is to extend this energy management system to incorporate more subsystems of the vehicle [3] or to consider emission constraints in the optimal control problem [4].

In the aforementioned approaches, the vehicle velocity (and thereby the power needed to propel the vehicle) is assumed to be fixed. Nevertheless, the vehicle inertia, which is the largest energy buffer in the vehicle, can have a large impact in energy savings and consequently in the extension of the driving range. For instance, in [5] it has been reported that changes in driving behavior could improve the energetic performance of the vehicle more than 30%. The promising improvements in energy efficiency have contributed to the emergence of the eco-driving concept, which aims to increase the energy efficiency of a vehicle by means of a convenient selection of driving strategies, i.e., legal regulations, technological implementations or simply changes in the driver behavior. Hence, it is clear that eco-driving is a broad concept where government, manufacturers and users participate [6].

To solve the eco-driving problem, standard techniques used in optimal control have been adopted, see [7] for a detailed overview of the recent literature. In [8], [9], dynamic programming (DP) has been used to find a global solution to this problem. Alternatively, Pontryagin’s minimum principle (PMP) has been used in [10]–[13]. The main disadvantages of PMP are that it only provides a necessary condition for optimality and that incorporating state constraints is not a simple task. Therefore, in [14]–[16] static nonlinear optimization techniques are used to solve the problem in the presence of state constraints. It remains unclear from the papers that use static optimization techniques or PMP whether the solutions are, in fact, globally optimal. Unfortunately, the literature related to this topic is scarce. The noticeable exception is [15], where the continuous-time optimal control problem is certified to be convex, which guarantees that the obtained solution is globally optimal. However, [15] does not discuss the possible loss of convexity due to the discretization process. This might occur, as it is demonstrated in this manuscript.

This paper aims to expose a detailed view of the global optimal solution to the eco-driving problem. The results of this paper can be used to certify optimality of the results presented in the existing literature and in future works. The main contributions presented in this paper are threefold: Firstly, a method to reformulate and discretize the problem is presented. This is initially done for a simplified case to illustrate the nonconvexity of the problem and subsequently extended to the complete eco-driving problem. Secondly, a detailed analysis of the uniqueness of the solution to the reformulated problem is used to obtain a set of mild conditions that guarantee the global optimality of the solution. Thirdly, a sequential quadratic programming (SQP) method is employed to efficiently solve the eco-driving problem.

II. CONTINUOUS-TIME PROBLEM FORMULATION

In this section, a continuous-time formulation of the eco-driving concept as an optimal control problem is provided. Eco-driving aims at obtaining an optimal control force $u(t)$ and velocity profile $v(t)$ that minimizes the integral of the power $P(v,u)$ consumed by a vehicle while traveling during a given time interval $[t_o,t_f]$ over a given trajectory $s(t) \in$
with known geographical characteristics, i.e., with a given road grade $\alpha : [s_0, s_f] \rightarrow [-\frac{\pi}{2}, \frac{\pi}{2}]$, and boundary conditions on position and velocity. This can be stated in the form of the following optimal control problem:

$$\min_{s(t), v(t), u(t)} \int_{t_0}^{t_f} P(v(t), u(t)) dt$$  \hspace{1cm} (1a)

subject to

$$m \frac{dv}{dt} = u(t) - f(v(t), s(t)), \hspace{1cm} (1b)$$

$$\frac{ds}{dt} = v(t), \hspace{1cm} (1c)$$

$$s(t_0) = s_0, \ t_k = s_f \hspace{1cm} (1d)$$

$$v(t) = v_o, \ t_k = v_f \hspace{1cm} (1e)$$

$$v \leq v(t) \leq v_o, \hspace{1cm} (1f)$$

where (1b) represents the longitudinal vehicle dynamics, in which $u(t)$ is the control force and $f(v, s)$ describes the aerodynamic drag, rolling resistance and gravity forces as $f(v, s) = \frac{1}{2} c_d \rho v^2 + c_r mg \cos(\alpha(s)) + mg \sin(\alpha(s))$. (2)

In (1b) and (2), $m$ represents the combined mass of the vehicle and the inertia of the driveline, $g \approx 9.81$ [m/s$^2$] is the gravitational constant, $c_r > 0$ describes the rolling force coefficient and $c_d = \frac{1}{2} \rho_d A_d$ with $c_d > 0$ is the drag coefficient, in which $\rho_d$ denotes the air density and $A_d$ is the frontal area of the vehicle.

The consumed power $P(v, u)$ can be obtained from different modeling concepts that capture the energy consumption in the powertrain. In this paper, it is assumed to be a quadratic function of the form

$$P(v, u) = \beta_0 v^2 + \beta_1 uv + \beta_2 u^2, \hspace{1cm} (3)$$

for some non-negative parameters $\beta_0, \beta_1$ and $\beta_2$. Equation (3) is a physically realistic approximation, e.g., for electric motors due to the fact that the friction and Ohmic losses are captured by the terms $\beta_0 v^2$ and $\beta_2 u^2$, respectively. The optimal control problem (1) does not consider constraints on the control force $u(t)$ nor constraints on propulsion power $P(v, u)$, as in (3). However, the results of this paper can be extended to include such constraints. Alternatively, constraints on propulsion power can be incorporated by connecting the eco-driving to vehicle energy management, as was done in [17],[18], where power constraints of powertrain components are present.

In general, (1) is a nonlinear optimal control problem that might have multiple local solutions due to specific features of the vehicle model and road profile, see (2). This implies that direct optimization methods or methods based on PMP only provide candidate minima, which might not correspond to the global solution to problem (1).

### III. Convexity in Relation to Discretization

To illustrate that the solution methods for the control problem (1) can introduce nonconvexity, which might complicate finding the global minimum, a simplified version of problem (1) is considered in this section. Using this simplified example, we will illustrate a possible reformulation of the problem as a convex optimal control problem, which implies the existence of a unique solution. Because of the presence of state constraints in (1), we will focus on discrete-time approximations of the optimal control, which might by itself introduce nonconvexity, even for specific cases where the continuous-time problem (1) is convex.

For a simplified version of problem (1) used in this section, consider $\beta_0 = 0$, $\beta_1 = 1$ and $\beta_2 = 0$ in (3), which corresponds to an electric motor with perfect energy conversion and no friction and Ohmic losses. Constant velocity bounds are also considered. Moreover, assume a flat road, meaning that $\alpha(s) = s$ so that the rolling friction is constant, i.e. $\cos(\alpha(s)) = 1$, and the gravitational force has no effect on the longitudinal vehicle dynamics as $\sin(\alpha(s)) = 0$. Under these assumptions, (1)-(3) reduces to

$$\min_{s(t), v(t), u(t)} \int_{t_0}^{t_f} u(t) v(t) \ dt. \hspace{1cm} (4a)$$

subject to

$$m \frac{dv}{dt} = u(t) - \sigma g v^2, \hspace{1cm} (4b)$$

$$v \leq v(t) \leq v_o, \hspace{1cm} (4c)$$

The above optimization problem is nonconvex due to (4b), meaning that application of PMP or finite dimensional optimization methods cannot guarantee a global optimal solution. We will show how this problem can be reformulated as a convex optimization problem, where we focus on discrete-time approximations so that static optimization methods can be applied.

#### A. Direct Discretization

In order to illustrate that caution should be taken when discretizing the optimal control problem (4) with (1c)-(1f), we show that direct discretization leads to a nonconvex optimization problem. In an attempt to assess convexity of (4) with (1c)-(1f), we eliminate $u(t)$ by substituting the equality constraints (4b) into the objective function (4a), thereby producing an equivalent optimization problem. This procedure is a useful technique to analyze the convexity of the problem in the feasible set, see, e.g., [18], and leads to

$$\min_{s(t), v(t)} \int_{t_0}^{t_f} \left( \frac{1}{2} m \frac{dv}{dt}^2 + c_r mg v(t) + \sigma g v(t)^3 \right) dt. \hspace{1cm} (5)$$

subject to (1c)-(1f).

In order to solve (5) subject to (1c)-(1f), using direct optimization methods, we can approximate the integral in the objective function using a forward Euler discretization method at $v_k = v(t_k)$ and $s_k = s(t_k)$, i.e., at instances $t_k$, $k \in \{0, \ldots, N\}$, for some $N \in \mathbb{N}$, where $t_{k+1} > t_k$, $t_0 = t_0$, $t_N = t_f$ and step size $\tau_k = t_{k+1} - t_k > 0$. This leads to a forward Euler discretization of the objective function in (5), given by

$$\sum_{k=0}^{N-1} \tau_k \left( \frac{m v_{k+1}^2 - m v_k^2}{2} + c_r mg v_k + \sigma g v_k^3 \right) \hspace{1cm} (6)$$

which can be rewritten as

$$\frac{1}{2} m \sum_{k=0}^{N-1} \tau_k (\sigma g v_k^3 + c_r mg v_k^2) - \frac{1}{2} m (v_{k+1}^2 - v_k^2)^2. \hspace{1cm} (7)$$

using (1e). The last term in this expression is nonconvex, which is a direct consequence of the forward Euler discretization. It should be noted that this result is independent of the step size $\tau_k$ and a similar conclusion can be drawn from the application of a backward Euler method. This shows the loss of...
convexity when applying discrete-time approximations of the eco-driving problem (1). Higher-order discrete approximations of (5) possibly lead to a convex optimization problem, albeit at the cost of increased complexity. Instead, we will here reformulate the optimal control problem (5) subject to (1c)-(1f) in a way that introduction of nonconvex terms is avoided, as will be shown below.

B. Continuous-Time Reformulation

The reformulation of the simplified eco-driving problem is based on the observation that, instead of discretizing the objective function directly, the first two terms in (5) can be integrated over the boundary conditions (1d) and (1e) to obtain

$$\min_{s(t),v(t)} \frac{1}{2} \beta_1 (v_2^2 - v_1^2) + c_m g (s_f - s_0) + \int_{t_0}^{t_f} \sigma_d v(t)^3 \, dt,$$  

subject to (1c)-(1f). The two terms in this objective function describe the change in kinetic energy of the vehicle over the complete trajectory, and the loss due to rolling resistance, respectively. Since $v_0$, $s_0$, $v_f$ and $s_f$ are known, these terms do not influence the optimal solution. Interestingly, the remaining term describes the energy losses due to aerodynamic drag. Thus, the reduced expression is discretized using a forward Euler approach, leading to

$$\min_{s_k,v_k} \sum_{k=0}^{N-1} \tau_k \sigma_d v_k^3,$$  

subject to $s_{k+1} = s_k + \tau_k v_k$, (1d)-(1f). The resulting discrete objective function (9) is convex for $v_k > 0$, which corresponds to the vehicle driving in forward direction.

Hence, this simplified case shows that a direct discretization of the eco-driving can lead to a nonconvex optimization problem, while a reformulated problem yields a convex optimization problem after discretization. The ideas in this section will be extended in the next section towards the complete eco-driving problem (1) to prove the existence of a unique solution of the optimal control problem.

IV. A GLOBAL SOLUTION TO THE ECO-DRIVING PROBLEM

In this section, the ideas presented in Section III will be applied to problem (1), without making the simplifications considered in Section III. This allows an equivalent optimization problem of reduced complexity to be formulated, which can be discretized using a forward Euler method without introducing additional nonconvex terms. The structure of the reduced discrete-time optimal control problem will be exploited to prove that a unique global optimal solution exists under mild and realistic conditions.

A. Reduction of the Continuous-Time Problem

Nonconvex optimization problems can show multiple local minima, therefore finding a global solution of the problem could be a cumbersome task. Nonconvexity is not only a consequence of a discretization action, it can also be related to other parameters of the nonlinear control problem (1).

For instance, (1a) could be a nonconvex objective function, which occurs for particular values of $\beta_1$, $\beta_2$, and $\beta_3$ that make (3) a nonconvex function. Moreover, a realistic road grade $\alpha(s)$ might also introduce nonconvexity in the equality constraint (1b). As done in the previous section, an equivalent optimal control problem will be obtained by substituting the equality constraint (1b) into the objective function.

Adopting the basic ideas presented in Section III, the continuous-time optimal control problem (1) is reformulated into a convenient form given by

$$\min_{v(t),s(t)} \int_{t_0}^{t_f} P(v(t), ma(t) + f(v(t), s(t))) \, dt,$$  

subject to (1c)-(1f) and

$$\frac{dv}{dt} = a(t),$$

where $a(t)$ is a new decision variable, which represents the vehicle acceleration, and $f(v, s)$ is given by (2). It is important to remark that (10a) is obtained from the substitution of (1b) into (1a), and its integrand is given by

$$P(v, ma + f(v, s)) = \beta_2 (mg \gamma_r(s) + c_v v^2 + c_m g \gamma_d(s) + ma)^2 + \beta_1 v (ma + c_v v^2 + mg \gamma_d(s) + c_m g \gamma_r(s)) + \beta_0 v^2,$$  

in which $\gamma_d(s) = \sin(\alpha)\gamma_d(s)$ and $\gamma_r(s) = \cos(\alpha(s))$. The relevance of (11) is that it contains information of the longitudinal vehicle dynamics, where the majority of the nonlinearities of the problem are embedded. This information and the structure of the problem (10) with (1c)-(1f) can be exploited to obtain a reduced, yet equivalent, optimal control problem.

The first step to achieve this goal is to observe that

$$\gamma_d(s) = \frac{dh}{ds} \text{ and } \gamma_r(s) = \frac{dh}{ds},$$

where $h(s)$ is the given elevation profile and $s'(t)$ is the horizontal projection of $s(t)$. The validity of these expressions can be explained using Fig. 1, where the geometry in this physical configuration shows that for any specific point in the road, the change in elevation and horizontal projection with respect to the displacement are given by $\frac{dh}{ds} = \sin(\alpha(s))$, and $\frac{dh}{ds} = \cos(\alpha(s))$, respectively.

The next step is to remove the terms in the objective function (10a) that can be solved in advance and have no contribution to the optimal control problem. In particular, consider the following terms from (10a):

$$E_G = \int_{t_0}^{t_f} \beta_2 m (a + g \gamma_d(s) + c_m g \gamma_r(s)) v + 2 \beta_2 m a \sigma_d v^2 \, dt,$$

$$= \int_{t_0}^{t_f} \beta_1 m \left( a \frac{dh}{ds} + \frac{dv}{dt} + g \frac{dh}{ds} + g \frac{dv}{dt} \right) + 2 \beta_2 m a \sigma_d v^2 \frac{dh}{ds} \, dt,$$

$$- \frac{1}{2} \beta_1 m (v_f^2 - v_0^2) + \beta_1 mg (h_f - h_0) + \beta_1 mg \gamma_r(s_f' - s_o') + \frac{3}{2} \beta_2 m a \sigma_d (v_f^2 - v_0^2).$$  

Fig. 1: Geometry of the road profile.
The first equality in (13) is obtained by substitution of (1c), (10b) and (12), and the second equality by solving the integral over the boundary conditions of the problem. The first two terms show that the total kinetic and potential energy of the vehicle depend only on the velocities and elevations at the boundaries. In a similar way, the last two terms demonstrate that part of the energy consumed by the drag and rolling forces, respectively, are defined by the velocity and horizontal displacement at the boundaries.

Since the value of (13) is given by the boundary conditions of the optimization problem (10), it is possible to rewrite (10a) as

\[
\int_{t_0}^{t_f} P(v, m a + f(v,s)) \, dt = \int_{t_0}^{t_f} P_R(a,v,s) \, dt + E_C, \tag{14}
\]

where

\[
P_R(a,v,s) = \beta_0 \sigma^2 + \beta_1 \sigma a^2 + 2 \beta_2 m a g(\gamma(s) + c_v \gamma(s)) + \beta_3 (m a)^2 + \beta_4 (m g \gamma(s)) + \sigma a^2 + c_v m g \gamma(s))^2. \tag{15}
\]

Removing the constant term \(E_C\) from the objective function, thereby changing (11) to (15) in the optimal control problem, does not change its optimal solution.

### B. Unique Solution to the Discrete-Time Optimal Control Problem

The reduced continuous-time optimal control problem obtained in the previous section can be discretized in order to make it solvable using static optimization methods. In this section, it will be shown that the discrete-time problem has a unique solution under realistic conditions and an efficient method to obtain this global minimum will be presented in Section V.

In order to discretize the problem, we consider again a forward Euler discretization method and define \(a_k = a(t_k)\), \(v_k = v(t_k)\) and \(s_k = s(t_k)\) at instances \(t_k = t_n +, n \in N\), with fixed step size \(\tau = \frac{t_f - t_0}{N}\), for some \(N \in N\). This leads to

\[
\begin{align*}
\min_{a_k, v_k, s_k} & \sum_{k=0}^{N-1} \tau P \mu_k(s_k, v_k) \quad \text{(16a)} \\
\text{subject to} & \quad s_{k+1} = s_k + \tau v_k, \quad v_{k+1} = v_k + \tau a_k, \quad s_0 = s_0, \quad \tau \geq 0 \quad \text{(16b)}
\end{align*}
\]

Theorem 1. Suppose optimization problem (16) is feasible. If \(\beta_2 > 0\) and if \(g \gamma'\left(\frac{m g \gamma(s_k)}{\beta_0 + \beta_1 \sigma a^2 + \beta_2 m a g(\gamma(s_k) + c_v \gamma(s_k))}\right) \neq 1\) for all \(k \in \{0, \ldots, N-1\}\), optimization problem (16) has a unique global minimum.

Proof. The first-order necessary conditions for optimality of (16) are given by the so-called Karush-Kuhn-Tucker (KKT) conditions, see, e.g., [18]. Instrumental in these KKT conditions is the Lagrangian

\[
L(a_k, s_k, v_k, \lambda_k, \mu_k, \nu_k) = \\
\sum_{k=0}^{N-1} \beta_0 \tau v_k^2 + \beta_1 \sigma \mu_k^2 + \beta_2 \tau (m a_k)^2 + \beta_3 (m g \gamma(s_k) + c_v m g \gamma(s_k))^2 + 2 \beta_4 m a g(\gamma(s_k) + c_v \gamma(s_k)) + \lambda_{k+1}(s_{k+1} - s_k - \gamma_v - \gamma_a) + \mu_k (v_k - \tau) + \nu_k (v_k - \tau) + \tau (v_k - v_k). \tag{17}
\]

Since all constraints are linear, a critical point, or stationary point, of (16) is characterized by

\[
\begin{align*}
\frac{\partial L}{\partial a_k} &= 2 \beta_2 \gamma' \left(\frac{m g \gamma(s_k)}{\beta_0 + \beta_1 \sigma a^2 + \beta_2 m a g(\gamma(s_k) + c_v \gamma(s_k))}\right) + \nu_k - \lambda_{k+1} = 0 \quad \text{(18a)} \\
\frac{\partial L}{\partial v_k} &= 2 \beta_3 m a g(\gamma(s_k) + c_v \gamma(s_k)) + 4 \beta_4 m a g(\gamma(s_k) + c_v \gamma(s_k)) + \lambda_{k+1} = \lambda_k = \lambda \tag{18b}
\end{align*}
\]

and \(\frac{\partial L}{\partial s_k} = 0\) described by (16b) and (16c) respectively for some Lagrange multipliers \(\mu_k, \lambda_k, \mu_k \geq 0, \lambda_k \geq 0, k \in \{0, \ldots, N\}, \) such that\( \mu_k (v_k - \tau) \) and \(\nu_k (v_k - \tau) = 0, \) (18d) and \(\lambda_k, \lambda_{k+1}, \lambda_{k+1} = \lambda \), and \(\lambda_{N+1}\) chosen such a way that (16d) and (16e) are satisfied. Now rewriting (18a) as

\[
\begin{align*}
\mu_k = \frac{1}{\beta_0 + \beta_1 \sigma a^2 + \beta_2 m a g(\gamma(s_k) + c_v \gamma(s_k))} \tag{19}
\end{align*}
\]

and substituting this into (18b) yields \(\lambda_k = \lambda_{k+1} - 2 \beta_3 m a g(\frac{1}{\beta_0 + \beta_1 \sigma a^2 + \beta_2 m a g(\gamma(s_k) + c_v \gamma(s_k))} + r_{k+1} = \lambda \tag{20}
\]

Substituting (19) into (16c) leads to a characterization of a critical point of (16), given by

\[
\begin{align*}
\lambda_{k+1} = \lambda_k + \frac{1}{\lambda_k} - \lambda \quad \text{(18d)}
\end{align*}
\]

and

\[
\begin{align*}
\Phi(s, v) &= \frac{2 \beta_3 m a g(\gamma(s_k) + c_v \gamma(s_k))}{\beta_0 + \beta_1 \sigma a^2 + \beta_2 m a g(\gamma(s_k) + c_v \gamma(s_k)) + \lambda_k} \tag{21c}
\end{align*}
\]

Furthermore, since the matrix \(\frac{1}{\lambda_k} - \lambda \) is full rank for every \(k \in \{0, \ldots, N-1\}\) by the hypothesis of the theorem, every pair \((\lambda_k, \lambda_{k+1})\) leads to a unique trajectory of \(\lambda_k, \lambda_{k+1}\), meaning that they can be uniquely chosen such that (16d) and (16e) are satisfied if (16) is feasible.

The KKT conditions provide necessary conditions for optimality in the sense that they characterize points that satisfy

\[
\nabla F(x) \cdot d = 0 \quad \tag{22}
\]

for all feasible directions \(d \) at \(x\). In the above equation \(\nabla F(x)\) is the gradient of the objective function (16a), where \(x = [a_0, \ldots, a_{N-1}, v_1, \ldots, v_N, s_1, \ldots, s_N, \mu_0, v_{N+1}, \tau, \lambda, \lambda_0, \lambda_1, \lambda_2]^{T}\), and the feasible directions \(d \in \mathcal{F}\) are all vectors that satisfy \(x + \alpha d \in \mathcal{F}\) for some \(\alpha > 0\) and where \(\mathcal{F} = \{x | (16b)-(16f)\}\), which is a compact set. Points satisfying (22) can in principle be minima, maxima or saddle points. Because of uniqueness
of the solutions to (21), the obtained critical point has to be a minimum. Indeed, suppose the critical point would be a maximum or a saddle point, there would exist at least one other \(x\) that would satisfy the necessary conditions. In other words, if \(x\) is a saddle point or a maximum, it is possible to move away from this \(x\) and lower the value of the objective function until the boundary of the feasible set \(\mathcal{F}\) would be reached. At this point, we would find another point \(x\) that satisfies (22). This contradicts the fact that there is only one critical point. Therefore, this unique critical point has to be the global minimum, which completes the proof.

Remark 1. The conditions presented in Theorem 1 are satisfied for many realistic cases. For instance, \(\beta_2 > 0\) is always satisfied if Ohmic losses of the electric motor are considered in the objective function. Moreover, standard road design guidelines, e.g., [19], suggest curvatures that yield \(\frac{\partial^2 y(x)}{\partial x^2} \ll 1\) and \(\frac{\partial y(x)}{\partial x} \ll 1\).

V. SOLUTION TO THE ECO-DRIVING PROBLEM

In this section, we will propose a solution method for solving the optimal control problem (16). The solution method we propose is Sequential Quadratic Programming (SQP). The effectiveness of SQP to solve constrained nonlinear optimization problems is the main reason for the large acceptance of this method. The concept of this approach is to iteratively solve linearly constrained quadratic (QP) subproblems, that are an approximation of the original problem evaluated in a previous solution, until some convergence criterion is achieved [20]. The uniqueness of the solution guarantees that the SQP algorithm finds the global minimum, provided that the algorithm converges.

Since the objective function in (16) is nonconvex even after the reformulation and reduction presented in Section IV, we employ the results of [21] to formulate a convex QP subproblem by making an approximation of the Hessian matrix of (15), which is given by

\[
H = \begin{bmatrix}
\frac{\partial^2 y}{\partial x^2} & \frac{\partial^2 y}{\partial x \partial v} & \frac{\partial^2 y}{\partial x \partial g} \\
\frac{\partial^2 y}{\partial x \partial v} & \frac{\partial^2 y}{\partial v^2} & \frac{\partial^2 y}{\partial v \partial g} \\
\frac{\partial^2 y}{\partial x \partial g} & \frac{\partial^2 y}{\partial v \partial g} & \frac{\partial^2 y}{\partial g^2}
\end{bmatrix},
\]

in which we have omitted the arguments of the function \(P_0(a, s, v)\) for compactness of notation, and where

\[
\begin{align*}
\frac{\partial^2 y}{\partial x^2} &= h_{11} = 2/2m^2, \\
\frac{\partial^2 y}{\partial x \partial v} &= h_{12}(s) = 2/2m^2 \left( \frac{\partial u_s(x)}{\partial x} + \frac{\partial v_s(x)}{\partial x} \right), \\
\frac{\partial^2 y}{\partial x \partial g} &= h_{13}(s,v) = 2/2m^2 \left( \frac{\partial u_g(x)}{\partial x} + \frac{\partial v_g(x)}{\partial x} \right), \\
\frac{\partial^2 y}{\partial v^2} &= h_{22}(s,v) = 2/(2m^2)(\frac{\partial u_s(x)}{\partial v} + \frac{\partial v_s(x)}{\partial v})^2 + (\sigma v + mgγ_s(s) + sγ_v(s)) \left( \frac{\partial^2 u_s(x)}{\partial v^2} + \sigma v + mgγ_s(s) \right), \\
\frac{\partial^2 y}{\partial v \partial g} &= h_{23}(s,v) = 4mg\gamma_s(s) \frac{\partial u_s(x)}{\partial v} + \frac{\partial v_s(x)}{\partial v}, \\
\frac{\partial^2 y}{\partial g^2} &= h_{33}(s,v) = 4β^2 + \frac{\partial v_g(x)}{\partial g}. 
\end{align*}
\]

(23)

Since \(h_{22}(s, v)\) and \(h_{33}(s, v)\) can become negative, we propose in this paper to use

\[
H(s, v) = \text{diag} \left( 2/2m^2, \epsilon_{22}, \max(h_{33}(s, v), \epsilon_{33}) \right),
\]

as a positive definite approximate Hessian in the SQP algorithm presented below, in which \(\epsilon_{22}, \epsilon_{33}\) are small positive numbers and \(h_{33}(s, v)\) is given by (24c).

The approximated Hessian matrix allows us to solve the eco-driving problem by sequentially solving the following convex second-order approximation of (15) as:

\[
\begin{align*}
\{a_{i+1}, s_{i+1}, v_{i+1}\} & \in K = \\
\arg \min_{a_i, s_i, v_i} & \sum_{k=0}^{N-1} \tau P_0(a_k, s_k, v_k, a_k', s_k', v_k') \cdot \\
\text{subject to} & \quad (16b)-(16f),
\end{align*}
\]

(25)

where

\[
P_0(a_k, s_k, v_k, a_k', s_k', v_k') = \frac{1}{2} \begin{bmatrix} s_k' \end{bmatrix}^T H(s_k, v_k) \begin{bmatrix} s_k' \end{bmatrix} + \begin{bmatrix} s_k' \end{bmatrix}^T \nabla P_0(a_k, s_k, v_k) - \frac{1}{2} \begin{bmatrix} s_k' \end{bmatrix}^T H(s_k, v_k) \begin{bmatrix} s_k' \end{bmatrix}.
\]

(26)

In (27), \(\nabla P_0(a_k, s_k, v_k)\) is the gradient of (15), and \(H(s_k, v_k)\) the positive definite approximated Hessian matrix given by (24), both evaluated at \((a_k', s_k', v_k')\), where \(i\) indicates the iteration of the SQP algorithm. Convergence of the proposed SQP approach is guaranteed due to the fact that the Hessian matrix (25) satisfies the conditions for convergence presented in [21, Section 3.2].

VI. NUMERICAL EXAMPLES

In this section, the SQP algorithm proposed in Section V is used to find the optimal solution of two numerical examples of the eco-driving problem.

A. Benchmark Problem for Electric Vehicles

In this example, we revisit the numerical benchmark problem for eco-driving that have been introduced in [10] and uses PMP to solve it. The energy consumption in this model considers a frictionless electric motor, i.e., \(\delta_0 = 0\). The effects of the rolling force in the vehicle are assumed to be constant, i.e., \(\gamma_v = 1\), while the effects of the gravitational force are described by

\[
\gamma(s) = g(\gamma_v s) = p_0 + p_1 s + p_2 s^2 + p_3 s^3.
\]

(28)

It is important to note that the polynomial function \(\gamma(s)\) also embeds information related to the road profile used in this example. In Table I, the parameters presented in [10] are translated to the eco-driving formulation proposed in this paper. Since the example of [10] is a numerical example, the parameters in Table I and the optimal solution do not have a physical interpretation. The method proposed in Section V returns an optimal solution that is depicted in Fig. 2a. This solution shows the same features reported in [10]. In fact, the final cost obtained using SQP is 1.2295 × 10^6, which differs only 0.0767% from the results reported in [10], while having a similar computation time. Considering that in this case \(\beta_2 > 0\), from Theorem 1, it is possible to conclude that the

| TABLE I: EV parameters in [10]. |
|-----------------|-----------------|-----------------|
| \(p_0\) | 3 | \(v(t)\) | 0 |
| \(p_1\) | 0.4 | \(v(t)\) | 0 |
| \(p_2\) | -1 | \(s(t)\) | 0 |
| \(p_3\) | 0.1 | \(s(t)\) | 10 |
| \(\sigma_d\) | 10^{-3} | \(v(t)\) | 0 |
Fig. 2: Global solution to the eco-driving problem [10].

Fig. 3: Global solution to the eco-driving problem.

solution is a global minimum of the problem. Using the SQP approach presented in this paper, velocity constraints can be easily added, as shown in Fig. 2b, which is not straightforward using the PMP-based approach of [10].

B. Hybrid Electric Heavy-Duty Vehicle

In this example, a hybrid electric heavy-duty vehicle driving at a constant speed is compared with an eco-driving strategy, which is depicted in Fig. 3 as a green surface.

The constant-speed driving profile is described by a solid line in Fig. 3. In this case, the total energy consumed by the vehicle is $3.0719 \times 10^4 [kJ]$. On the other hand, the dashed line presented in Fig. 3 describes the optimal control path and velocity profile obtained as the global solution to the eco-driving optimal control problem studied in this case. The total energy consumed by the vehicle under this strategy is $2.843 \times 10^4 [kJ]$, which is approximately 7.44% lower than the energy consumed by the vehicle driving at constant velocity.

TABLE II: Parameters for the heavy-duty vehicle example.

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<th>Parameter</th>
<th>Value</th>
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<tbody>
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<td>$k_0$</td>
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<tr>
<td>$k_1$</td>
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<tr>
<td>$k_2$</td>
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<tr>
<td>$\tau$</td>
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</tr>
<tr>
<td>$v(t_f)$</td>
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</tr>
<tr>
<td>$v_{\text{avg}}$</td>
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</tr>
<tr>
<td>$\beta$</td>
<td>80 [km/h]</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>60 [km/h]</td>
</tr>
</tbody>
</table>

VII. CONCLUSIONS

In this paper, a detailed study has been conducted on the global optimality of the eco-driving optimal control problem. We have proposed to reformulate and discretize the problem and have subsequently derived conditions that guarantee the existence of the global solution to the eco-driving problem. Taking advantage of these results, a SQP algorithm that efficiently solves the eco-driving problem has been proposed. The methodologies and results were illustrated in two numerical examples.

REFERENCES


A Shrinking Horizon Approach to Eco-driving for Electric City Buses: Implementation and Experimental Results

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Abstract: Eco-driving aims at minimizing the energy consumption of a vehicle by changing the vehicle’s velocity. This can be formulated as an optimal control problem and this paper presents an efficient shrinking horizon implementation to solve this problem. The efficient implementation is demonstrated on a case study of a fully electric bus, driving on an inner-city public transport route. Because the bus drives on designated bus lanes, meaning that it has little interaction with other traffic, and because it has frequent and predictable stops, this case will have a good energy consumption savings potential. An energy consumption reduction of 11.43% is achieved on a simulation study for the case that the vehicle is fully autonomous and a reduction of 6.94% is achieved experimentally for the case that the driver is ‘coached’ using a driver assistance system.

Keywords: Automotive Control, Advanced Driver Assistance Systems, Eco-driving

1. INTRODUCTION

Electrification of vehicles, and particularly the deployment of fully electric vehicles, is considered as a way to mitigate the production of CO₂ emissions from fossil fuels (Grauers et al., 2012). However, electric vehicles are known to have limited range and to charge slowly, when compared to traditional vehicles, leading to range anxiety, which is the user’s concern to not reach the vehicle’s destination (Nilsson, 2011). There are several factors that influence range anxiety, e.g., the limited energy storage capacity of batteries, the ability to accurately predict the vehicle’s energy consumption and the total energy consumption of the vehicle (including its auxiliaries). The last item mentioned can be improved using vehicle energy management (de Jager et al., 2013; Onori et al., 2016) or using eco-driving strategies.

Eco-driving intends to change the vehicle’s speed between departure and arrival, either by assuming full automation of the vehicle or by changing the driver’s behavior (Bingham et al., 2012; Scarretta et al., 2015). This problem can be formulated as an optimal control problem (OCP). To solve this problem, standard techniques from optimal control are often adopted. In (Ozatay et al., 2014), Dynamic Programming (DP) has been used to solve the eco-driving problem. Alternatively, Pontryagin’s Minimum Principle (PMP) has been used in (Dib et al., 2014; Ozatay et al., 2017; Kamal et al., 2011). The main disadvantage of DP is that it is computationally intensive and the main disadvantage of PMP is that incorporating state constraints is not a simple task. Therefore, static nonlinear optimization techniques are used in (Padilla et al., 2018; Romijn et al., 2018; Murgovski et al., 2015; Vajedi and Azad, 2016) to solve the problem in the presence of state constraints. These static optimization problems lead to an optimization problem that is often seen in model-predictive control and allows for fast online implementations.

While many of the existing results on eco-driving present only simulation results (Kamal et al., 2013; Ojeda et al., 2017; Vajedi and Azad, 2016) and/or present a one-shot optimal solution when the full drive trajectory is known (Padilla et al., 2018; Ozatay et al., 2014; Murgovski et al., 2015), this paper presents an online shrinking horizon implementation of eco-driving, and experimental results that shows its potential to reduce the energy consumption. As a case study, a fully electric city-bus is used on an inner-city public transport route. The considered city bus drives on designated lanes, meaning that it has limited interaction with other traffic, and has frequent and predictable stops. Therefore, a good energy savings potential is expected. First, an efficient online implementable algorithm of the solution strategy of (Padilla et al., 2018) is presented. In this efficient algorithm, the solution to the state-space model is substituted into the objective function and constraints, as is often done in model-predictive control, leading to a significant reduction of the number of decision variables, when compared to (Padilla et al., 2018). This implementation can also be used in a so-called shrinking horizon fashion. A shrinking horizon is needed because at every time instant, the optimal control input is recomputed for a shorter time horizon, due to the fact that the time and distance to the next bus stop is smaller than during the previous time instant. We will present two results: the first being a simulation study for the case...
that the vehicle speed is exactly controlled (i.e., assuming a fully automated vehicle), and the second being an experimental study for the case that the driver is ‘coached’ using a driver assistance system that interacts with the driver through a human-machine interface. Both studies will show that the energy consumption of the bus can be reduced significantly.

2. ECO-DRIVING CONTROL PROBLEM

In this section, we introduce the continuous-time eco-driving problem and the discrete-time solution approach presented in (Padilla et al., 2018). This solution approach will serve as a basis for our proposed efficient real-time implementation.

2.1 Continuous-Time Formulation

The eco-driving problem considered in (Padilla et al., 2018) aims at minimizing the total energy consumed by a vehicle over a given time interval [t_k, t_f] and trajectory s(t) ∈ [s_0, s_f] for a road grade α(s) ∈ [−z, z], which depends on the position s. This can be formulated as a continuous-time Optimal Control Problem (OCP) given by

\[
\begin{align}
\min_{s(t), v(t)} & \quad \int_{t_0}^{t_f} P(t, u(t)) dt \\
\text{subject to} & \quad m \ddot{s}(t) = u(t) - \sigma_a v^2 + mg\gamma(s), \\
& \quad \dot{s}(t) = v(t), \\
& \quad g = s_0, \\
& \quad v(0) = v_0, \\
& \quad v(t_f) = v_f, \\
& \quad \gamma \leq v(t) \leq \bar{v}, \\
& \quad \bar{v} \leq u(t) \leq \bar{a},
\end{align}
\]

where (1b) represents the longitudinal vehicle dynamics of a vehicle with mass m, aerodynamic drag coefficient \(\sigma_a\) and rolling resistance and gravity forces, with gravitational constant \(g\), where the latter two are combined in \(\gamma(s) = c_r \cos(\alpha(s)) + \sin(\alpha(s))\), (2)

where \(c_r\) describes the rolling resistance coefficient. Furthermore, the non-negative velocity bound is bounded by (1f). In (1g), we have also included bounds on the acceleration, which can be done without compromising the uniqueness of the solution claimed by (Padilla et al., 2018). Finally, the consumed power in the driveline is assumed to be a quadratic function given by

\[
P(v, u) = \beta_0 v^2 + \beta_1 vu + \beta_2 u^2,
\]

where \(\beta_0\), \(\beta_1\) and \(\beta_2\) are non-negative parameters that describe the contribution of Ohmic and mechanical friction losses in the power consumption.

2.2 Discretization

In (Padilla et al., 2018), the OCP (1) is discretized at times \(t_k = k\tau + t_0, k \in K = \{0, \ldots, N\}\), with time steps \(\tau = \frac{t_f - t_0}{N-1}\), for some \(N \in \mathbb{N}\) using a forward Euler discretization method. Additionally, the non-linear equality constraint (1b) is included in the objective function (1a), resulting in a Quadratic Programming (QP) problem, given by

\[
\min_{s_k, v_k, u_k} \sum_{k=0}^{N-1} \tau P(s_k, v_k, u_k)
\]

subject to

\[
\begin{align}
& s_{k+1} = s_k + \tau v_k, \\
& u_{k+1} = u_k + \tau \alpha_k, \\
& s_0 = s_0, \\
& v_0 = v_0, \\
& s_N = s_f, \\
& v_N = v_f, \\
& \gamma \leq v_k \leq \bar{v}, \\
& \bar{v} \leq u_k \leq \bar{a},
\end{align}
\]

where \(a_k = a(t_k)\), \(v_k = v(t_k)\) and \(s_k = s(t_k)\), and

\[
P(s_k, v_k, u_k) = \beta_0 v_k^2 + \beta_1 vu_k + \beta_2 u_k^2 + mg\gamma(s_k)
\]

represents the driveline power consumption, now dependent on the acceleration, position and velocity of the vehicle.

2.3 Sequential Quadratic Programming for Eco-driving

The uniqueness and global optimality of the solution to the discrete-time OCP (4) has been formally shown by (Padilla et al., 2018). Moreover, it was suggested that Sequential Quadratic Programming (SQP) is a suitable approach to obtain the solution. In SQP, a nonlinear program is solved by iteratively solving QP approximations of the nonlinear program. For the OCP (4), this leads to

\[
\{a^{i+1}, x^{i+1}\} = \arg \min_{a, x} \sum_{k=0}^{N} \tau P_QP(a, x, a^i, x^i)
\]

subject to (4b) – (4d), for all \(k \in K\), with \(P_QP(a, x, a^i, x^i)\), a second-order approximation of (5), given by

\[
P_QP(a, x, a^i, x^i) = \frac{1}{2} a^T H a + \left[ G_a (a - a^i) + a^i - x^i )^T H (a - a^i) \right]
\]

where the superscript \(i\) refers to the solution of the \(i\)-th iteration of the SQP algorithm, and

\[
a = [a_0, \ldots, a_{N-1}]^T \in \mathbb{R}^N,
\]

\[
x = [x_0, v_0, \ldots, x_{N-1}, v_{N-1}]^T \in \mathbb{R}^{2N},
\]

\[
H = \text{diag}(H_a, H_g) \in \mathbb{R}^{N \times N},
\]

\[
G_a = \left[ \frac{\partial P(a, x, u)}{\partial a_n} \right]_{n=0}^{N-1}, \ldots, \left[ \frac{\partial P(a, x, u)}{\partial a_{N-1}} \right]_{n=0}^{N-1} \in \mathbb{R}^{N \times N},
\]

\[
G_x = \left[ \frac{\partial P(a, x, u)}{\partial x_n} \right]_{n=0}^{N-1}, \ldots, \left[ \frac{\partial P(a, x, u)}{\partial x_{N-1}} \right]_{n=0}^{N-1} \in \mathbb{R}^{N \times N},
\]

In (10), the Hessian matrix is approximated to a diagonal, positive definite matrix (Padilla et al., 2018), given by

\[
H_a = 2 \beta_3 m^2 I_N \in \mathbb{R}^{N \times N},
\]

\[
H_g = \text{diag}(c_r, m, \ldots, c_r, m),
\]

in which \(I_N \in \mathbb{R}^{N \times N}\) is an \(N\)-dimensional identity matrix, \(c_r\) and \(m\) are small positive numbers that guarantee the positive definiteness of the Hessian matrix, and
The gradients of the power consumption (5) are considered in (11) and (12), where
\[ \frac{\partial P_{\text{tot}}}{\partial s_k} = 3 \left( \frac{m g}{s_k} + 3 \sigma_{a} v_k^2 \right), \]
\[ \frac{\partial P_{\text{tot}}}{\partial x_k} = -3 \left( \frac{m g}{s_k} + 3 \sigma_{a} v_k^2 \right), \]
\[ \frac{\partial P_{\text{tot}}}{\partial v_k} = 6 \left( \frac{m g}{s_k} + 3 \sigma_{a} v_k^2 \right). \]

The QP iterations (6) repeat until its solution converges. We consider the algorithm to have converged if
\[ \left\| a^i - a^{i-1} \right\|_2 \leq \rho, \]
where \( \left\| \cdot \right\|_2 \) is the 2-norm operator, \( i \) and \( i-1 \) represent the current and previous QP solutions, respectively, and \( \rho \in \mathbb{R} \) is a given non-negative convergence tolerance.

This approach provides a tractable solution to the eco-driving OCP (4). In the next section, we will modify this method to obtain a real-time implementation for the eco-driving problem.

### 3. REAL-TIME IMPLEMENTATION

To arrive at a real-time implementable algorithm, the computation time needs to be low, such that solutions can be obtained in a shorter time than the sampling period used in the embedded system. In this section, we will modify the SQP method presented in the previous section to reduce the computation time by means of a reduction on the number of decision variables and the application of a shrinking horizon approach that uses warm-start for the next optimization problem. This reduction of the number of decision variables does not affect the solution (and its optimality).

#### 3.1 Improving Computational Efficiency

In this section, we will reduce the number of decision variables by eliminating the state \( x \) in (6). This leads to a significant reduction of the computation time, since the computation time of the QP subproblems scales cubically with the number of decision variables (Diehl et al., 2009).

By eliminating the state \( x_k \) in \( [s_k, v_k]^{\top} \) from the QP problems an improvement of the computation time is expected by a factor of 3\(^3 = 27 \), when compared to (Padilla et al., 2018).

In order to reduce the number of decision variables, the longitudinal motion of the vehicle described by the equality constraints (4b)-(4d) can be recast as a prediction model for instants \( k \in \mathbb{K} \) as
\[ x = \Phi x_0 + \Gamma n, \]
where \( \Phi \in \mathbb{R}^{2N \times 2} \) and \( \Gamma \in \mathbb{R}^{2N \times N} \) are defined by
\[ \Phi = \begin{bmatrix} I_2 & A^2 \\ \vdots & \vdots \\ I_{2N-2} & A_{2N-2} \end{bmatrix}, \quad \Gamma = \begin{bmatrix} 0 \quad 0 \\ B \quad 0 \\ \vdots & \vdots \\ B_{2N-2} \quad 0_{2N-2} \end{bmatrix}, \]
with
\[ A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \end{bmatrix}. \]

and \( \Phi \), a \( n \)-dimensional column vector with all entries zero. For \( k = N \), the final state given by (4e) can be expressed as the following equality constraint
\[ A_N^{-1}B \quad A_N^{-2}B \quad \ldots \quad B \quad a = x_N - A_N x_0. \]

By substituting (10) and (20) into (7), we remove the dependency on the state \( x \) in (7). After removing the constant terms (which do not change the solution), we obtain
\[ \text{PQP}(a, a') = \frac{1}{2} a^{\top} \left( \hat{H}_a + \Gamma^\top \hat{H}_a^\top \right) a \]
\[ + \left( G_a^\top - a^\top \hat{H}_a + \left( G_a^{(1)} - a^\top \Gamma^\top \hat{H}_a \right) \Gamma \right) a, \]
where \( \hat{H}_a \) and \( \hat{H}_a^\top \) are the approximated Hessian respect to acceleration and state, respectively, evaluated in the \( i \)-th iteration of the SQP algorithm.

Using the above notation, we obtain the following QP problem with acceleration \( a \) as the only decision variable
\[ \{ a^{i+1} \} = \text{argmin}_{a} \sum_{k \in \mathbb{K}} \tau \text{PQP}(a, a') \]
subject to (23),
\[ v - \Xi \phi x_0 \leq \Xi \Gamma a \leq \bar{v} - \Xi \phi x_0, \]
\[ 1_{N,2} \leq a \leq 1_{N,2}, \]
where the constraint (25b) is obtained by substituting (20) in (4f) for all \( k \in \mathbb{K} \) and considering \( v = \Xi x \), with a selection matrix \( \Xi = 1_{N} \otimes [1 \ 1] \in \mathbb{R}^{N \times 2N} \), in which the operator \( \otimes \) denotes the Kronecker product. Besides, the constraint (25c) is directly obtained from (4g) for all \( k \in \mathbb{K} \) and considering \( 1_{N} \in \mathbb{R}^{n} \), a \( n \)-dimensional column vector with all entries one. The QP subproblem (25) is sequentially solved until it converges, thus finding the global optimal solution to the discrete optimal control eco-driving problem (4), with the corresponding convergence criteria defined by
\[ \| a^{i} - a^{i-1} \|_2 \leq \rho. \]

In practice, a smaller number of decision variables let the optimization problem converges with fewer iterations, which might lead to an additional reduction in computation time.

#### 3.2 Considerations for Real-time Implementation

In order to achieve further improvements in terms of computation time, we use an efficient QP solver with warm-start and a shrinking horizon approach. Shrinking horizon works in a similar way as receding horizon, also known as model-predictive control. At every time instant \( k \), the full optimization problem (25) is solved over the horizon \( N \) using the actual state \( x_k \) as initial value. This leads to a sequence of optimal control inputs \( a \), where only the first \( a_0 \) is implemented at time instant \( k \). Now at time \( t_{k+1} \) the process repeats, but in contrary to receding horizon control, the control horizon \( N \) now becomes 1 time step shorter. The use of shrinking horizon for eco-driving of buses is motivated by the fact that we only need to optimize the vehicle speed until the next bus stop and at every recomputation of the optimal control inputs, the next bus stop becomes nearer. The fact that the control horizon shrinks makes the optimization problem time dependant. In particular, at time instant \( k \), the eco-driving problem has to be solved...
over the discrete-time set $K^k = \{k, \ldots, N\}$. This causes the QP subproblem at time $k$ to become
\[
\min_{\alpha^{k+1}} \sum_{k \in K^k} \gamma \, P_{QP}(\alpha^k, \alpha^{k+1})
\]
subject to (23), (25b), (25c)

where $N$ has become $N-k$ in (23) and the matrices in (25b) and (25c) are modified similarly. The Hessian and gradient matrices in the SQP algorithm also need to be modified mutatis mutandis. The shrinking of the horizon is repeated until $N-k < \min_{\alpha^{max}}$ for some $\min_{\alpha^{max}}$ to avoid unnecessary computations in the last iterations, where not much energy consumption improvement is expected.

The QP problem (27) is implemented in Simulink, with the solver mpccpsolver.m, from the Matlab/Simulink Model Predictive Control Toolbox. This functionality allows to compile the Simulink model as a system function that can embed the eco-driving problem and provide the solution in real-time. This solver can also make a so-called warm-start to reduce the computation time of the solution by providing the optimal solution of the previous iteration of the algorithm as an initial guess for the next iteration.

4. SIMULATION STUDY FOR ELECTRIC CITY BUSES

Inner-city public transportation buses, sometimes drive through exclusive lanes to arrive and depart from stops at well defined times. This makes this scenario to have high predictability, which makes it highly suitable for the implementation of eco-driving approaches.

In this section, we will analyze the performance of the solution to eco-driving problem, presented in Section 3.1, in the context of electric city buses. The first part of this section shows an identification approach to obtain a control-oriented model for the power consumption of the driveline in an electric bus, which can be used in the real-time optimal ecodriving problem of Section 3. Later, this model is used to obtain an optimal velocity profile for a specific driving route and finally, ideal energy savings are calculated with respect to a benchmark velocity profile, that has been obtained from real data logged from a bus driving on a segment of 2.5[km] between two bus stops, where the elevation between the highest and the lowest point is less than one meter. Hence, the grade $\beta(s)$ is considered zero for all $s$.

4.1 Identification of the Power Consumption Model

The OCP (4) considers a simplified control-oriented power consumption model defined by (5). For the identification of this model, several realizations over the 2.5[km] route mentioned above are used. This leads to a set of velocity profiles $\{v_j\}$, where $j \in J = \{1, \ldots, J\}$ represents the $j$-th realization of the velocity profile. These velocity profiles are then simulated on a high-fidelity (hi-fi) vehicle model to obtain power consumption profiles $\{P_{hi-fi}(s)\}$ for each given velocity profile. This data can be used to find the parameters $\beta_0$, $\beta_1$ and $\beta_2$ in (5) by solving the following constrained least-squares problem
\[
\begin{align*}
&\min_{\beta_0, \beta_1, \beta_2} \sum_{j \in J} \sum_{k \in K^k} \| P_{hi-fi}(s, v_j) - \hat{P}_k(s) \|_2^2 \\
&\text{subject to } \beta_0 > 0, \beta_1 > 0 \text{ and } \beta_2 > 0,
\end{align*}
\]
where $P_{hi-fi}(s, v_j)$ is defined in (5). It is important to remark that the hi-fi vehicle model is a validated model.

Fig. 1(b) depicts the normalized vehicle power consumption, based on a real velocity profile that is shown in Fig. 1(a). The power consumption described by the hi-fi model is represented by the dotted red line, while the blue line is obtained by identifying the three unknown parameters on the simplified consumption model (5).

Although the identified model shows a considerable error with respect to hi-fi model, it is capable to describe a general trend in its behavior. In the last part of this section, we will show that this feature is enough to produce accurate solutions in terms of energy savings.

4.2 Computation Time

The identified model obtained in the previous section is used to obtain the optimal velocity profile for an electric bus driving on the 2.5[km] flat route. The optimal velocity profile is obtained by the method proposed in (Padilla et al., 2018) and the approach proposed in this paper. The computation time for both cases is compared in Table 1. It should be noted that the computation time presented in (Padilla et al., 2018) is obtained using a different QP solver and cannot be compared to the results presented in Table 1, where we have compared the method presented in this paper with (Padilla et al., 2018) using the same QP solver.

As it was stated in Section 3.1, the improved computation time obtained by the approach proposed in this paper is mainly produced by the reduced number of decision variables. The results shown in Table 1 demonstrate that the proposed implementation can be used in real-time applications.
### 4.3 Energy Savings

In order to analyze the performance of the method proposed in this paper, in terms of energy savings, a real velocity profile is used as benchmark. In Fig. 2, the blue dashed line depicts a real velocity profile obtained from logged data of an electric bus, while the red solid line represents the optimal velocity profile obtained as solution to (27). By using the hi-fi model we obtain 11.43% of energy savings, while using the simplified function (5), we save 12.21% of energy. This shows that the simplified power consumption model (5) is an adequate approximation that enables an efficient calculation of velocity profiles and provides a good approximation of the expected energy savings.

The power profile of this benchmark is represented as a red dashed line on Fig. 1(b). Here, it is possible to see the on-off action created by the real behavior of a driver. Although the velocity profile generated by the driver seems to be close to the optimal velocity profile presented in Fig. 2, the constant on-off control action created by the driver produces energy losses that explains the energy savings when applying the optimal velocity profile.

### 5. ASSISTED DRIVING EXPERIMENTAL RESULTS

In this section, we will discuss the experimental results obtained by using a driving assistance system that ‘coaches’ the driver to follow the optimal velocity profile obtained as a solution to the eco-driving problem (27). First, we provide details about the case study analysed in this section. Second, the Human Machine Interface (HMI) used to provide visual feedback to the driver is described; and finally, experimental results for the case study are provided and discussed.

#### 5.1 Case Study

The experiments have been conducted in the previously selected road, with a distance of 2.5[km] that should be covered in 200[s], considering a real speed limit of 60[km/h]. During the experiments, it was registered wind velocity of 30[km/h], approximately matching the road direction.

The baseline for energy savings is generated by the driver covering the selected road, with the knowledge of velocity bounds and initial and final states. This is compared to the energy consumed while providing visual feedback to the driver and expressed as a percentage ratio of energy savings.

Two drivers have performed the experiments during one day, driving the same bus in both directions. The power consumption and velocity profiles have been recorded with a sample rate of 1[Hz] for data post-processing.

#### 5.2 Human-Machine Interface

The Human-Machine Interface (HMI) provides visual feedback to the driver at a specific frequency, based on the solution to the real-time implementation of the eco-driving problem (27). The HMI updating frequency should take into account the speed of reaction of the driver. In practice, this means that the update frequency presented to the driver is considerably smaller than the one used for the real-time implementation of (27).

In this case, an external computer manages the HMI with an update frequency of 0.1[Hz]. It has been developed in Vector CANoe (Vector Informatik GmbH, 2016) and depicted in Fig. 3. The HMI sends the request to increase (Fig. 3(a)), hold (Fig. 3(b)) or decrease (Fig. 3(c)) the current velocity, in order to follow the optimal velocity profile within a tolerance of ±1[km/h].

#### 5.3 Experimental Results

In this section, we will experimentally determine the amount of energy that eco-driving is able to save. Subsequently, we will compare this results with simulations in the hi-fi model with the velocity profiles generated experimentally. While performing the experiments, a wind velocity of 30[km/h] was registered, with a direction approximately matching the selected road. Hence, the experimental eco-driving energy savings are separated for headwind and tailwind driving, and shown in Table 2. In average, we obtained 6.94% of energy savings.

Note that the percentages in Table 2 were obtained from 20 experiments performed by the two drivers, i.e., four experiments for each wind direction, plus the corresponding baselines. The velocity profiles of two experiments with its...
Table 3. Simulated energy savings

<table>
<thead>
<tr>
<th>Wind direction</th>
<th>Driver</th>
<th>Energy savings</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Headwind</td>
<td>Driver 1</td>
<td>6.48%</td>
<td>6.89%</td>
</tr>
<tr>
<td></td>
<td>Driver 2</td>
<td>7.29%</td>
<td></td>
</tr>
<tr>
<td>Tailwind</td>
<td>Driver 1</td>
<td>14.42%</td>
<td>15.96%</td>
</tr>
<tr>
<td></td>
<td>Driver 2</td>
<td>17.90%</td>
<td></td>
</tr>
<tr>
<td><strong>Total Average</strong></td>
<td></td>
<td><strong>11.43%</strong></td>
<td></td>
</tr>
</tbody>
</table>

baselines are depicted in Fig. 4, one for each driver with the specified wind direction. Here, the blue dotted line represents the baseline and the red line shows the driver behavior when receiving eco-driving feedback.

Tailwind driving saves more energy than headwind. In this case, the difference is approximately a factor of three (see Table 2). The difference in energy savings between the drivers can be explained by their own driving style and experience, as well as random disturbances, e.g., changes in intensity and direction of the wind, temperature variations, traffic conditions, etc.

The experimental velocity profiles were logged by Vector CANoe and used as input to the hi-fi vehicle model. Using the same baseline for each driver and wind direction, the simulated energy savings are shown in Table 3. Tables 2 and 3 show correlation between experimental and simulated energy savings, verifying that the hi-fi vehicle model has been validated. Note that in both cases the eco-driving solution to (27) reduces the energy consumption.

6. CONCLUSIONS

This paper has presented an efficient shrinking horizon implementation to solve the eco-driving OCP. The implementation has been demonstrated on a case study of a fully electric bus, driving on an inner-city public transport route. We have shown that the computational performance can be reduced significantly using the proposed implementation. Furthermore, a energy consumption reduction of 11.43% is achieved on a simulation study for the case that the vehicle is fully autonomous and a reduction of 6.94% is achieved experimentally for the case that the driver is ‘coached’ using a driver assistance system. The obtained results are promising in terms of energy savings by providing feedback to the driver and, at the same time, it has become evident that better results can be achieved by improving the accuracy when following the eco-driving feedback.

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A Distributed Optimization Approach for Complete Vehicle Energy Management

T.C.J. Romijn, M.C.F. Donkers, J.T.B.A. Kessels, and S. Weiland

Abstract—In this paper, a distributed optimization approach is proposed to solve the complete vehicle energy management problem of a hybrid truck with several controllable auxiliaries. The first part of the approach is a dual decomposition, which allows the underlying optimal control problem to be solved for every subsystem separately. For the second part of the approach, the optimal control problem for every subsystem is further decomposed by splitting the control horizon into several smaller horizons. Two methods for splitting the control horizon are used; the first method uses Alternating Direction Method of Multipliers and divides the horizon a priori, while the second method divides the horizon iteratively by solving unconstrained optimization problems analytically. We demonstrate the approach by solving the complete vehicle energy management problem of a hybrid truck with a refrigerated semi-trailer, an air supply system, an alternator, a DC/DC converter, a low-voltage battery and a climate control system. Simulation results show that the fuel consumption can be reduced up to 0.52% by including smart auxiliaries in the energy management problem. More interestingly, the computation time is reduced by a factor of 64 up to 1825, compared with solving a centralized convex optimization problem.

Index Terms—Energy Management, Optimal Control, Distributed Optimization, Hybrid Vehicles, Smart Auxiliaries

I. INTRODUCTION

Reducing fuel consumption has been one of the top priorities of the automotive industry in the last decades. A low fuel consumption is important for a sustainable society and is also one of the largest competitive factors for sales. Over the last decades, several new technologies have been introduced in vehicles to improve fuel efficiency. A major improvement is obtained with the introduction of hybrid technology. By adding an electric motor with a high-voltage battery to the powertrain, braking energy can be recuperated and the internal combustion engine can be controlled at a more efficient operating point. The energy management strategy in this case is restricted to determining the (optimal) power split between the electric motor and the internal combustion engine. The energy consumption is, however, not only related to the driving functionality of the vehicle, especially for heavy-duty vehicles, as part of the energy is used for auxiliaries, e.g., an air supply system, a refrigerated semi-trailer and a climate control system. These auxiliaries have a potential energy buffer that can be utilized by the energy management system to schedule energy flows and thereby further improving the vehicle energy efficiency, i.e., Complete Vehicle Energy Management (CVEM, see [1]). At the same time, heavy-duty vehicles are augmented with many different type of auxiliaries, which requires the energy management system to be flexible and scalable with low complexity to reduce development time and costs. Moreover, as heavy-duty vehicles typically drive long distances, it is needed to solve the CVEM problem over large horizons to demonstrate and benchmark the benefit of CVEM.

Many different solution strategies for solving the energy management problem have been proposed over the past decades. The proposed solution strategies can be divided into so-called online and offline solution strategies [2], [3]. Online solution strategies are real-time implementable and can be further divided in rule-based strategies (see, e.g., [4]), equivalent consumption minimization strategies (ECMS, see, e.g., [5]) or model predictive control (MPC, see, e.g., [6]). The online solution strategies only rely on feedback and/or predictions and can therefore not guarantee the global optimal solution. To verify the performance of the online solution strategies and to analyze different configurations, so-called offline solution strategies have been developed based on, e.g., dynamic programming (DP, see, e.g., [7]–[9]), Pontryagin’s minimum principle (PMP, see, e.g., [10]–[12]) or convex optimization (see, e.g., [13], [14]). The offline solution strategies require all disturbances to be known (e.g., the driving cycle) so that the global optimal solution can be computed and can therefore not be implemented in real-time. Still, they do provide a benchmark for online solution methods and are therefore valuable tools. We focus on offline solution strategies in this paper.

While online optimization methods based on ECMS might be able to handle the complexity of the CVEM problem, the aforementioned offline optimal control methods cannot. It should be noted that multi-state energy management problems, e.g., including battery state-of-health [15], battery aging [16], thermal management [17]–[19] and the control of a waste heat recovery system [20] are all based on ECMS, meaning that global optimality of the solution cannot be guaranteed. For the offline optimization methods, scalability is poor as the computational complexity of DP increases exponentially with the number of states and solving the two-point boundary value problem resulting from PMP is difficult, particularly when state constraints are present, see, e.g., [17] in the context of thermal dynamics. Finally, a convex approximation of the
energy management problem can lead to a globally optimal solution, but still requires a large-scale optimization problem to be solved.

For this reason, distributed solutions for energy management start to appear. In [21], [22], an online implementable game-theoretic approach to CVEM is shown. In [23], scalability is obtained by using the Alternating Direction Method of Multipliers (ADMM) while ideas based on ECMS are used to calculate the equivalent costs at a supervisory level. Still, these distributed solutions are all online solution methods for which the global optimal solution is not guaranteed.

In this paper, we propose to use methods from distributed optimization to solve the convex approximation of the CVEM problem and obtain the global optimal solution which can be used to verify the performance of online solution methods and to analyze the potential of different auxiliaries for energy management. In particular, we use the dual decomposition approach for CVEM that we first introduced in [24] in combination with efficient algorithms to solve the dual functions that we first introduced in [25]. Dual decomposition has been used in large-scale optimization since the early 1960s [26] and have since then been applied to problems with large scale dynamical systems (see, e.g., [27]–[29]).

With the dual decomposition approach [24], the optimal control problem is decomposed into smaller dual problems. Each of the dual problems is then either solved explicitly, with an ADMM algorithm or solved with a Lagrangian Method [25]. Both papers showed good performance on a simplified CVEM problem which considered only an internal combustion engine, an electric motor, a high-voltage battery and a refrigerated semi-trailer while real scalability of the method with the number of components has not been demonstrated. Contrary to [24] and [25], we present the general CVEM problem as a quadratically constrained linear program (QCLP), where we apply a relaxation to ensure convexity of the CVEM optimal control problem. This allows the method to be applied to various vehicle configurations. Moreover, the dual decomposition approach is extended with a Newton dual update algorithm to improve convergence speed. The ADMM algorithm is extended for multi-state dynamical systems and the Lagrangian Method is extended to handle systems with saturated states on the lower or upper bound. Finally, to fully demonstrate the performance of the distributed optimization approach, the optimal control of a hybrid truck with an internal combustion engine, an electric motor, a high-voltage battery, a refrigerated semi-trailer, an air supply system, an alternator, a DCDC converter, a low-voltage battery and a climate control system is solved. Moreover, the proposed algorithm is compared with the state-of-the-art solver CPLEX [30].

The remainder of this paper is organized as follows. The general optimal control problem and the application of the dual decomposition is given in Section II. In Section III, solution methods are presented to solve the dual functions that resulted from the dual decomposition. The CVEM problem is casted as an optimal control problem in Section IV and, finally, simulation results are presented in Section V.

II. DISTRIBUTED OPTIMIZATION OF POWER NETS

In this section, we consider the optimal control of the energy flows in a power net, which is illustrated in Fig. 1. The power net consist of energy storage devices, e.g., a high-voltage battery, and energy converters, e.g., an electric machine. The storage devices are connected to the converters, while the outputs $y_{m,k}$ and inputs $u_{m,k}$ of the converters are connected to each other via nodes according to a specific topology, i.e., energy can be exchanged directly between converters but not directly between storage devices. At each node $n \in \{1, \ldots, N\}$, there is also a known exogenous load signal $v_{n,k}$ given for each time instant $k$. Subsystems are composed of a combination of a converter, possibly with an energy storage device. The goal of the power net is to minimize the cumulative energy losses of all subsystems, while meeting constraints on the inputs, outputs and states in each subsystem. In this section, we will introduce the optimal control problem for this power net and we will give a dual decomposition approach to solve the optimal control problem. In Section IV we will show that the Complete Vehicle Energy Management (CVEM) problem can be represented as a power net, where minimizing the energy losses is equivalent to minimizing the fuel consumption.

A. Optimal Control Problem

The optimal control problem for the power net is given by

$$\min_{\{u_{m,k}, y_{m,k}\}} \sum_{m \in M} \sum_{k \in K} c_{m} u_{m,k} - d_{m} y_{m,k}, \quad (1a)$$

where $u_{m,k} \in \mathbb{R}$ and $y_{m,k} \in \mathbb{R}$ are the (scalar) inputs and outputs of the converter in subsystem $m \in M = \{1, \ldots, M\}$ with $M$ the number of subsystems and at time instant $k \in K = \{0, 1, \ldots, K - 1\}$, with $K$ the horizon length. In (1a), $c_{m} \in \mathbb{R}$ and $d_{m} \in \mathbb{R}$ are coefficients that define the energy losses in converter $m$. Moreover, we use the notation $\{u_{m,k}, y_{m,k}\}$ to indicate $\{u_{m,k}, y_{m,k}\} \in M \times K$. This notation will be used throughout the paper for minimizing over a set. The optimization problem (1a) is to be solved subject to a quadratic equality constraint describing the input-output behavior of each converter, i.e.,

$$\frac{1}{2} y_{m,k} v_{m,k}^2 + f_{m,k} u_{m,k} + e_{m,k} + y_{m,k} = 0, \quad (1b)$$
with \( q_{m,k} \in \mathbb{R}_+ \), \( f_{m,k} \in \mathbb{R} \) and \( e_{m,k} \in \mathbb{R} \) being efficiency coefficients of the converter \( m \in \mathcal{M} \) at time instant \( k \in \mathcal{K} \), and subject to linear system dynamics of the storage device in subsystem \( m \in \mathcal{M} \), i.e.,

\[
x_{m,k+1} = A_m x_{m,k} + B_m u_{m,k} + B_m u_{m,k},
\]

for all \( k \in \mathcal{K} \), where the initial state \( x_{m,0} \) and final state \( x_{m,K} \) of the storage device are assumed to be given, \( u_{m,k} \) is a known load signal at every time instant \( k \) and the input \( u_{m,k} \in \mathbb{R} \) is subject to linear inequality constraints, i.e.,

\[
\mathbf{w}_{m,k} \leq u_{m,k} \leq \mathbf{w}_{m,k},
\]

for all \( k \in \mathcal{K} \) and \( m \in \mathcal{M} \), and the state \( x_{m,k} \) is subject to linear inequality constraints, i.e.,

\[
x_{m,k} \leq x_{m,k} \leq \mathbf{w}_{m,k},
\]

for all \( k \in \mathcal{K} \) and \( m \in \mathcal{M} \). Finally, the optimization problem is solved subject to a linear equality constraint describing the interconnection of the subsystems, i.e.,

\[
\sum_{m \in \mathcal{M}} A_m u_{m,k} + B_m y_{m,k} + \frac{1}{M} y_k = 0,
\]

for all \( k \in \mathcal{K} \), where \( A_m \in \mathbb{R}^{n \times n} \) and \( B_m \in \mathbb{R}^{n \times N} \) are vectors with the \( n \)-th element being \(-1\) if the power flow to node \( n \) is positive, \( 0 \) if there is no power flow to node \( n \) and \( 1 \) if the power flow to node \( n \) is negative. Here, \( N \) is the number of nodes in the power system where power is aggregated. Furthermore, the load signal \( v_k = \{v_{1,k}, \ldots, v_{N,k}\}^T \in \mathbb{R}^N \) is assumed to be known at each time instant \( k \in \mathcal{K} \). We define the primal optimal solution as the solution \( \{w_{m,k}^*, y_{m,k}^*, \mu_k^*\} \) that satisfies (1c) and (1e) as the primal optimal value of (1). Note that quadratic behavior (1b) is a good assumption for a wide range of converters as most converters have quadratic power losses, e.g., the internal combustion engine or the electric machine, which will be shown in Section IV.

**B. Dual Decomposition and Convex Relaxation**

The optimization problem (1) can be a large-scale problem (when \( K \) and \( M \) are large), which is not convex due to the quadratic equality constraint (1b). We propose in this paper to solve (1) by decomposing it into several smaller problems and to relax (1b). In doing so, we can solve (1) efficiently without sacrificing optimality of the solution as we will show below. Problem (1a) subject to (1b) - (1f) cannot be separated due to the complicating constraint (1f). Therefore, we decompose the problem via dual decomposition by introducing the following so-called partial Lagrangian

\[
L(\{u_{m,k}, y_{m,k}, \mu_k\}) = \sum_{m \in \mathcal{M}} \sum_{k \in \mathcal{K}} C_{m,k} u_{m,k} - d_{m,k} y_{m,k} + \mu_k^T (A_m u_{m,k} + B_m y_{m,k} + \frac{1}{M} y_k),
\]

where \( \mu_k \in \mathbb{R}^N \) is a Lagrange multiplier, subject to (1b)- (1e). Indeed, the partial Lagrangian is obtained by adding the complicating constraints (the constraints that act on more than one subsystem) to the objective function in (1a). The partial Lagrange dual function is now given by

\[
g(\{\mu_k\}) = \min_{\{u_{m,k}, y_{m,k}\}} L(\{u_{m,k}, y_{m,k}, \mu_k\}) = \sum_{m \in \mathcal{M}} g_m(\{\mu_k\}),
\]

with

\[
g_m(\{\mu_k\}) = \min_{\{u_{m,k}, y_{m,k}\}} \sum_{k \in \mathcal{K}} C_{m,k} u_{m,k} - d_{m,k} y_{m,k} + \mu_k^T (A_m u_{m,k} + B_m y_{m,k} + \frac{1}{M} y_k),
\]

subject to (1b) - (1e). Note that each of the Lagrange dual functions (3b) subject to (1b) - (1e) is related to one of the subsystems and can be solved independently. The dual problem is given by

\[
\max_{\{\mu_k\}} g(\{\mu_k\}) = d^*,
\]

subject to (1b) - (1e) where \( d^* \) is defined as the dual optimal value. The dual problem (4) gives a lower bound on the primal optimal value \( p^* \) of problem (1), i.e.,

\[
d^* \leq p^*.
\]
\[ L(\{u_{m,k}, y_{m,k}, \mu_k\}) = \sum_{m \in M} \sum_{k \in K} c_{m,u_{m,k}} - d_{m,y_{m,k}} + \mu_k^T (A_m u_{m,k} + B_m y_{m,k} + \frac{1}{2} y_{m,k} y_{m,k}) + \nu_{m,k}(y_{m,k} + \frac{1}{2} q_{m,k} u_{m,k}^2 + f_m k u_{m,k} + c_{m,k}), \]  

subject to (1c)-(1e) and where \( \nu_{m,k} \geq 0 \) is the (scalar) Lagrange multiplier associated with the (scalar-valued) quadratic inequality constraint (1b'). The derivative with respect to \( y_{m,k} \) at \( \{u_{m,k}^*, y_{m,k}^*, \mu_k^*\} \) of this partial Lagrangian (6) (one of the necessary conditions for optimality, see, e.g., [31]) is given by

\[ \begin{align*}
B_m^T \mu_k^* - d_m + \nu_{m,k}^* &= 0. 
\end{align*} \]

Since \( d_m - B_m^T \mu_k^* \geq 0 \) by the hypothesis of the theorem, it follows that \( \nu_{m,k}^* \geq 0 \). The positivity of \( \nu_{m,k} \) ensures that inequality (1b') is satisfied as an equality by complementarity slackness [31] which completes the proof.

The first condition of this theorem is in general not mild but provides an additional check for the dual optimal solution \( \{u_{m,k}^*, y_{m,k}^*, \mu_k^*\} \) that satisfies (4) with \( \{\mu_k\} \) defined in (3) with the minimum taken over (1c)-(1e) and (1b') instead of (1b), to be equal to the primal optimal solution to (1). Moreover, for the objective function (1a) this condition is intuitively satisfied as the energy losses in each subsystem \( m \in M \) are defined as \( c_{m,u_{m,k}} - d_{m,y_{m,k}} \) for each time instant \( k \in K \), so that the quadratic inequality constraint (1b') at the optimal solution implies

\[ c_{m,u_{m,k}} - d_{m,y_{m,k}} \geq d_{m,k} u_{m,k}^2 + f_m k u_{m,k} + c_{m,k} \]

for all \( m \in M \) and \( k \in K \). If the quadratic inequality constraint holds with equality at the optimal solution for all \( m \in M \) and \( k \in K \), then the energy losses in each subsystem \( m \in M \) are minimal, which is intuitively needed for the second condition to be optimal. The second condition in the hypothesis of this theorem is relatively mild (Slater’s constraint qualification, see, e.g., [31]) and can be satisfied for the numerical example given in Section IV. The dual problem (4) can be solved efficiently using a subgradient method as will be shown below.

### C. Maximizing the Lagrange Dual Function

Maximizing the Lagrange dual function (3) over \( \mu_k \) can be done with a 'steepest ascent' method,\[ \mu_k^{s+1} = \mu_k^s + \alpha_k^s \left( \sum_{m \in M} A_m u_{m,k}^s + B_m y_{m,k}^s + \frac{1}{2} y_{m,k}^s y_{m,k}^s \right), \]

for all \( k \in K \) where \( \alpha_k^s \) is a suitably chosen matrix and \( s \in N \) is the iteration counter. In [24], a diagonal matrix with sufficiently small positive constant step sizes on its diagonal was chosen such that the Lagrange dual problem will always converge but convergence tended to be slow. A better convergence rate is achieved with a Newton scheme (see e.g., [32]). We will derive this scheme intuitively from a primal feasibility perspective. The idea is to update the dual variables \( \mu_k \) such that for the next iteration primal feasibility for the complicating constraints holds, i.e.,

\[ \sum_{m \in M} A_m u_{m,k} + B_m y_{m,k} + \frac{1}{2} y_{m,k} y_{m,k} = 0. \]

Furthermore, we can approximate the value for \( u_{m,k}^{s+1} \) and \( y_{m,k}^{s+1} \) linearly by

\[ \begin{align*}
&u_{m,k}^{s+1} \approx u_{m,k}^s + \left( \frac{\partial u_{m,k}^s}{\partial \mu_k} \right)^T \left( \mu_k^{s+1} - \mu_k^s \right), \\
y_{m,k}^{s+1} \approx y_{m,k}^s + \left( \frac{\partial y_{m,k}^s}{\partial \mu_k} \right)^T \left( \mu_k^{s+1} - \mu_k^s \right),
\end{align*} \]

where \( \frac{\partial u_{m,k}^s}{\partial \mu_k} \) is a vector with the approximations of the first-order derivatives of \( u_{m,k}^s(\mu_k) \) with respect to the dual variables \( \mu_k \) at iteration \( s \). Similarly, \( \frac{\partial y_{m,k}^s}{\partial \mu_k} \) is a vector with the approximations of the first-order derivatives of \( y_{m,k}^s(\mu_k) \) with respect to the dual variables \( \mu_k \) at iteration \( s \). By substituting (11) into (10) and solving for \( \mu_k^{s+1} \) we obtain (9) with

\[ \alpha_k^s = \left( \sum_{m \in M} A_m \left( \frac{\partial u_{m,k}^s}{\partial \mu_k} \right)^T - B_m \left( \frac{\partial y_{m,k}^s}{\partial \mu_k} \right)^T \right)^{-1}, \]

which can be obtained by calculating the vector with derivatives for each subproblem in a distributed fashion. Note that calculating the derivatives with respect to \( \mu_k \) in (11) can be hard and might not even exist due to the presence of constraints. Instead, the derivatives can be approximated by neglecting the state constraints (1e). This might cause that (9) does not converge. In this case, sufficiently small constant step sizes can be chosen as was done in [24]. Convergence speed might be significantly slower in this case, as will be shown in the simulation study in Section V. Finally, the dual decomposition algorithm consists of iteratively solving (3) to obtain \( \{u_{m,k}^*, y_{m,k}^*, \mu_k^*\} \) and updating the Lagrange multipliers by solving (9) to obtain \( \{\mu_k^{s+1}\} \). In the Section below, we will provide methods to efficiently solve the dual functions (3b).

### III. EVALUATING THE DUAL FUNCTIONS

Each of the Lagrange dual functions (3b) related to one of the subsystems can be solved separately and can be written as a linearly constrained quadratic program (LCQP) by substituting (1b) into (3b), which gives

\[ g_m(\{\mu_k\}) = \min_{\{u_{m,k}\}} \sum_{k \in K} \frac{1}{2} H_m k u_{m,k}^2 + F_m k u_{m,k} + E_m k, \]

with

\[ \begin{align*}
&H_m = (d_m - B_m^T \mu_k) q_{m,k}, \\
&F_m = c_m + A_m \mu_k + (d_m - B_m^T \mu_k) f_{m,k}, \\
&E_m = \mu_k^T \mu_k + (d_m - B_m^T \mu_k) c_{m,k}
\end{align*} \]

and subject to (1c) - (1e). Note that for strict convexity of (13a), it is required that \( d_m - B_m^T \mu_k > 0 \).
this condition is satisfied, as will be shown with the numerical example in Section V.

As a result, the dual decomposition allows solving the quadratically constrained quadratic program by solving multiple LCQPs iteratively. However, solving a LCQP for a large horizon length $K$ is still very inefficient. Therefore, we introduce two solution methods to solve the optimization problem (13a), related to each of the subproblems, efficiently.

Both methods use the principle of splitting the horizon $K$ into multiple smaller intervals, where each interval is defined as $K_{\ell} = \{K_{\ell, 1}, \ldots, K_{\ell} - 1\}$ with $0 = K_0 < K_1 < \ldots < K_L = K$ and where $\ell \in L = \{1, \ldots, L\}$ with $L$ the number of intervals. To decompose the constraints in the optimization problem (13a) into smaller optimization problems, we recall that a solution to (1c) satisfies

$$x_{m,k,1} = \tilde{A}_{m,k,1} - x_{m,k,1} + \sum_{i \in \mathcal{K}_{(N_i, \ldots, k)}} A_{m}(B_{m,0}w_{m,i} + B_{m,n}u_{m,i}), \quad (14a)$$

for all $k \in K_{\ell}$, where the local initial condition $x_{m,K_{\ell} - 1}$ at each interval $\ell$ is equal to the final condition at interval $\ell - 1$, i.e.,

$$x_{m,K_{\ell} - 1} = \tilde{x}_{m,K_{\ell} - 1},$$

(14b)

for $\ell \in L$ and $\tilde{x}_{m,0} = x_{m,0}$ and $\tilde{x}_{m,L} = x_{m,K}$ which follow from the initial and final condition of the full horizon. Using these constraints, we can write the dual function (13a) as

$$g_m = \min_{(u_{m,k}, x_{m,K_{\ell} - 1})} \sum_{k \in K_{\ell}} \sum_{i \in \mathcal{K}_{(N_i, \ldots, k)}} \frac{1}{2} H_{m,k}u_{m,k}^2 + F_{m,k}u_{m,k} + E_{m,k},$$

subject to (1d), (1e) and (14). Note that the problem (15) subject to (1d), (1e) and (14) is only coupled by (14b).

We will introduce two solution methods that can be used to select the intervals $K_{\ell}$ and the initial state at each interval $\tilde{x}_{m,K_{\ell} - 1}$. In the first solution method, based on Alternating Direction Method of Multipliers (ADMM), the horizon is split a priori in a fixed number of intervals. For each interval, an optimization problem is solved that takes the initial state $\tilde{x}_{m,K_{\ell} - 1}$ as decision variable.

ADMM is a suitable method for the resulting decomposed problem is not strictly convex, yet still convex, as we will show below. In the second solution method, based on the Lagrangian Method, the horizon is split iteratively and the initial state is fixed on the lower or upper state constraint depending on the solution of the state-unconstrained optimization problem. The Lagrangian Method is only applicable to systems with scalar states, while the ADMM method is applicable to systems with multiple states.

### A. Horizon Splitting with ADMM

For this method, we define a priori the sets $K_{\ell} = \{K_{\ell, 1}, \ldots, K_{\ell} - 1\}, \ell \in L = \{1, \ldots, L\}$. This method is similar to the method proposed in [33] where intervals that contain only one time instant, i.e., $K_{\ell} = \{\ell - 1\}$ are used for solving the problem over a short horizon. Contrary to [33], we use intervals containing multiple time instants, thereby making it more applicable for solving the problem over a long horizon as will be demonstrated with the numerical example in Section V. The objective function in (15) is separable in variables related to each interval but is not strictly convex due to the minimization over the local initial state $\tilde{x}_{m,K_{\ell} - 1}$, which is an essential assumption for the dual decomposition approach taken in the previous section. Lagrangian methods as used in Section II.B, however, assume convexity of the objective function rather than strict convexity. Instead, the partial augmented Lagrangian for problem (15) can be defined as

$$L(\{u_{m,k}, x_{m,K_{\ell} - 1}, u_{m,\ell}\}) = \sum_{\ell \in K_0} \sum_{k \in K_{\ell}} \frac{1}{2} H_{m,k}u_{m,k}^2 + F_{m,k}u_{m,k} + E_{m,k} + u_{m,\ell}(\tilde{F}_{x_{m,K_{\ell} - 1} - x_{m,K_{\ell} - 1}}) + \frac{1}{2} R_{x_{m,K_{\ell} - 1} - x_{m,K_{\ell} - 1}},$$

(16)

in which $u_{m,\ell} \in \mathbb{R}^{dim(u_{m,\ell})}$ are Lagrange multipliers and where $R > 0$ is a diagonal matrix with penalty parameters on its diagonal. In this expression, we temporarily omit the constraints that are acting only within one interval, i.e., (1d), (1e) and (14a) and will reintroduce them later in the decomposed problem. The partial augmented Lagrange dual function is given by

$$\bar{g}_m(\{u_{m,l}\}) = \min_{(u_{m,k}, x_{m,K_{\ell} - 1})} L(\{u_{m,k}, x_{m,K_{\ell} - 1}, u_{m,\ell}\})$$

$$= \min_{\ell \in L} g_m(\{u_{m,\ell - 1}, u_{m,\ell}\}),$$

(17a)

with

$$\bar{g}_m(\{u_{m,\ell - 1}, u_{m,\ell}\}) = \min_{(u_{m,k}, x_{m,K_{\ell} - 1})} \sum_{k \in K_{\ell}} \frac{1}{2} H_{m,k}u_{m,k}^2 + F_{m,k}u_{m,k} + E_{m,k},$$

$$+ \frac{1}{2} R_{x_{m,K_{\ell} - 1} - x_{m,K_{\ell} - 1}} x_{m,K_{\ell} - 1} + \tilde{F}_{x_{m,K_{\ell} - 1} - x_{m,K_{\ell} - 1}} + \tilde{E}_{u_{m,k} - u_{m,k}},$$

(17b)

in which

$$\tilde{G} = \tilde{G}_{x_{m,\ell} - x_{m,K_{\ell} - 1} - R - \tilde{u}_{m,\ell}} A_{m,K_{\ell} - K_{\ell} - 1},$$

(17c)

$$\tilde{F} = F_{m,k} - \tilde{u}_{m,\ell} A_{m,K_{\ell} - K_{\ell} - 1} B_{m,n},$$

(17d)

$$\tilde{E} = E_{m,k} - \tilde{u}_{m,\ell} A_{m,K_{\ell} - K_{\ell} - 1} B_{m,u},$$

(17e)

for $\ell \in L$, with $u_{m,L} = 0$ and is to be solved subject to (1d), (1e) and (14a). Expressions (17c,d,e) are obtained by substituting (14a) for $k = K_{\ell} - 1$ into (16), only for the linear part of the equation. This gives a more desirable expression for (17c), i.e., the Hessian matrix is diagonal. However, as a result, (17c) is not separable due to the term $x_{m,K_{\ell} - 1}$. In (17c), by minimizing (17) sequentially from interval $\ell = 1$ to interval $\ell = L$, as part of the ADMM algorithm (see, e.g., [31]), the minimization problem (17), can still be solved efficiently.

To maximize the partial augmented Lagrange dual function, we use a ‘steepest ascent’ method, i.e.,

$$u_{m,\ell}^{k+1} = u_{m,\ell}^k + R(u_{m,\ell}^k - x_{m,K_{\ell} - 1}),$$

(18)

with $k \in \mathbb{N}$ the iteration number and for some given initial condition $u_{m,\ell}^{k,0}$, for $\ell \in L$. Finally, the ADMM algorithm consists of iteratively solving (17) subject to (1c) - (1e) to obtain $u_{m,\ell}^k$ for $\ell \in L$, $k \in K$ and solving (14a) for $k = K_{\ell} - 1$ to obtain $x_{m,K_{\ell} - 1}^k$ for $\ell \in L$, $k \in K$, followed by an update of the Lagrange multipliers through (18) to obtain $u_{m,\ell}^{k+1}$ for $\ell \in L$. 

D3.5 – Report on driving range prediction and extension algorithm

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B. Horizon Splitting with the Lagrangian Method

Fixing the interval \( K_\ell \ a \ priori \) and use ADMM to solve (15) results in a general solution method to solve the LCQP (13). We will also develop an iterative procedure that involves splitting the intervals based on solving (15) for the particular case where \( x_{m,k} \in \mathbb{R} \), i.e., the energy storage device in subsystem \( m \) is a scalar-state system. In Section IV, it will be shown that many components in the CVEM problem can be represented by a scalar-state system and in Section V we will show that a tailored solution method for these components is more favorable with respect to computation time, which also emphasizes the advantage of using the dual decomposition approach to CVEM where each of the dual functions can be solved with the most suitable solution method.

For this method, we initially take only one interval, i.e. the full horizon, so that \( K_\ell = \{ K_{\ell-1}, \ldots, K_{\ell-1} \} = \{ 0, \ldots, K-1 \} \) and \( \ell \in L = \{ 1 \} \) and solve (13a) subject to (1d) and (14) without considering the state constraints (1e). The main reason for this is that the problem without (1e) is much easier to solve. Depending on the solution of a state-constrained optimization, extra intervals will be added as will be shown later in this section.

First, we define the state unconstrained problem for subsystem \( m \), which is given by (15) subject to (1d) and (14). The Lagrangian of this problem is given by

\[
L((u_{m,k}, \lambda_k, \tau_{m,k}, v_{m,k})) = \sum_{k \in K_\ell} \sum_{k \in K_{\ell-1}} \frac{1}{2} H_{m,k} u_{m,k}^2 + F_{m,k} u_{m,k}^m_k + E_{m,k} + \nu_k(\tau_{m,k} - \tau_{m,k}) + \lambda_k \left( \frac{A_{m,K}^{K-1} - \tau_{m,k} - \tau_{m,K}^{K-1}}{\nu_k} \right) + \nu_k \left( \frac{A_{m,K}^{K-1} - \tau_{m,k} - \tau_{m,K}^{K-1}}{\nu_k} \right)
\]

with \( \lambda_k \in \mathbb{R} \), the Lagrange multiplier associated with the constraint (14), \( \nu_k \in \mathbb{R} \) and \( \lambda_k \in \mathbb{R} \), the Lagrange multipliers associated with the upper and lower input constraints (1d), respectively. The Karush-Kuhn-Tucker conditions [31] for minimizing the Lagrangian in (19) are given by the first-order necessary optimality condition

\[
\partial L(u_{m,k}, \lambda_k, \tau_{m,k}, v_{m,k}) = H_{m,k} u_{m,k} + F_{m,k} u_{m,k}^m_k + \lambda_k \left( \frac{A_{m,K}^{K-1} - \tau_{m,k} - \tau_{m,K}^{K-1}}{\nu_k} \right) = 0
\]

for all \( k \in K_\ell, \ell \in L \), feasibility of the constraint (14) and the complementary slackness conditions for the inequality constraints

\[
\tau_{m,k} = 0, \quad \nu_k(\tau_{m,k} - \tau_{m,k}) = 0
\]

for all \( k \in K_\ell, \ell \in L \) with \( \nu_k \geq 0 \) and \( \nu_k \geq 0 \). Finding a solution for (20) and (21) simultaneously is difficult and often the solution is found with a shooting method and a bisection algorithm over \( \lambda_k \), leading to the optimal solution

\[
u_k = -H_{m,k}^{-1}(F_{m,k} + \lambda_k \frac{A_{m,K}^{K-1} - B_{m,K}}{\nu_k})
\]

for all \( k \in K_\ell, \ell \in L \) for a given \( \lambda_k, \tau_{m,k} \) and \( \nu_k \). Instead, we propose a procedure that aims, for each interval \( \ell \in L \), at solving

\[
\lambda_k^{\ell+1} = (1 - \gamma)\lambda_k^\ell + \gamma H_{m,k}^{-1}(A_{m,K}^{K-1} - B_{m,K} + \nu_k(\tau_{m,k} - \tau_{m,K}^{K-1}))
\]

with relaxation parameter \( \gamma \in (0, 1) \) and with

\[
\nu_k = \max\left\{ 0, -H_{m,k} u_{m,k} - F_{m,k} - \lambda_k \right\}
\]

\[
\lambda_k^{\ell+1} = \max\left\{ 0, -H_{m,k} u_{m,k} - F_{m,k} - \lambda_k \right\}
\]

\[
\nu_k = \max\left\{ 0, -H_{m,k} u_{m,k} - F_{m,k} - \lambda_k \right\}
\]

with \( \ell \in \mathbb{N} \) the iteration index and for \( \lambda_k^\ell = 0 \) and \( \nu_k^\ell = 0 \) for all \( k \in K \), until (14) is satisfied within some desired tolerance. The expressions in (23) are obtained by substituting (22) into (14) and (1d). If \( H_{m,k} \) is strictly positive and if there exists an optimal solution \( u_{m,k}^* \) for which (1d) and (14) are satisfied, then the solution of (23) will converge to the solution of (15) subject to (1d) and (14) for a well chosen relaxation parameter \( \gamma \in (0, 1) \).
if \( x_m, K_{m-l} = \tilde{x}_m, K_{m-l} \) and \( x_m, K_l = \tilde{x}_m, K_l \), and if either one of the following conditions holds for all \( k \in K_l \):

- \( A_m > 0, B_m, u < 0, H_m, k_u w_{m, k+1} - H_m, k_u w_{m, k} < 0 \), \( F_{m, k+1} - F_{m, k} < 0 \), and \( F_{m, k+1} - F_{m, k} < 0 \) for all \( k \in K_l \) such that the first-order optimality conditions and the complementary slackness conditions are satisfied and \( \tilde{u}_{m, k} \) is optimal for all \( k \in K_l \).

This theorem provides a priori verifiable conditions when the optimal state trajectory is saturated at the lower bound or upper bound for all \( k \in \{ K_{l-1}, \ldots, K_l - 1 \} \), respectively. The three preceding results, i.e., i) the solution of the optimal control problem without considering the state constraints, ii) the iterative method for splitting the control problem into smaller ones to incorporate state constraints and iii) conditions for which the optimal solution satisfies \( x_m, k = \tilde{x}_m, k \) or \( x_m, k = \bar{x}_m, k \) for all \( k \in K_l \) allow us to propose the following algorithm for solving (13).

**Algorithm 1.** Take \( K_l = \{ 0, \ldots, K_l - 1 \} \) and let \( \hat{x}_{m,0}, \hat{x}_m, k \) be given.

- For each interval \( \ell \in \mathcal{L} \), check if the conditions of Theorem 2 are satisfied.
  - If the conditions of Theorem 2 are satisfied, the optimal solution satisfies (24a) or (24b).
  - If the conditions of Theorem 2 are not satisfied, compute the input constrained solution using (23). Then verify

\[
\ell_{\mathcal{L}} = \max_{k \in K_l} \{ \tilde{x}_m, k - x_m, k \}, \quad (29a) \quad \tilde{m}_l = \max_{k \in K_l} \{ x_m, k - \tilde{x}_m, k \}. \quad (29b)
\]

- If \( \ell_{\mathcal{L}} > 0 \) and \( \tilde{m}_l > \eta_l \), the lower state constraint is violated more than the upper state constraint and

\[
\hat{K}_l = \arg \max_{k \in \mathcal{K}_l} \{ x_m, k - \tilde{x}_m, k \}. \quad (29c)
\]

is added to the set of contact points \( \{ K_l \}_{\ell \in \mathcal{L}} \) and re-ordered, i.e., \( 0 = K_0 < \cdots < K_{l-1} < K_l \). To define new subsets \( K_{l-1} = \{ K_{l-1}, \ldots, K_l - 1 \} \) and \( x_m, K_{l-1} = \tilde{x}_m, K_{l-1} \). If \( \eta_l > 0 \) and \( \eta_l > \tilde{m}_l \), the upper state constraint is violated more than the lower state constraint and

\[
\hat{K}_l = \arg \max_{k \in \mathcal{K}_l} \{ x_m, k - \tilde{x}_m, k \}. \quad (29d)
\]

is added to the set of contact points \( \{ K_l \}_{\ell \in \mathcal{L}} \) and re-ordered, i.e., \( 0 = K_0 < \cdots < K_{l-1} < K_l \). To define new subsets \( K_{l-1} = \{ K_{l-1}, \ldots, K_l - 1 \} \) and \( x_m, K_{l-1} = \tilde{x}_m, K_{l-1} \). If both (29b) and (29a) are nonpositive, the \( \ell \)-th interval does not have to be further divided.

**Repeat until** \( \max \{ x_m, k - \tilde{x}_m, k, x_m, k - \bar{x}_m, k \} \leq 0 \) for all \( k \in \mathcal{K}_l \).

Similarly as the dual decomposition allows the large-scale optimal control problems to be solved by solving smaller optimal control problems on subsystem level, Algorithm 1 and the ADMM algorithm allow the optimal control problem over a large horizon to be solved through multiple optimal
control problems over a smaller horizon. Note that, to ensure convergence of the solution to the dual problem (4), the solution to each dual function obtained with Algorithm 1 or ADMM (18) needs to be converged before proceeding with the maximization in (9). Still, by combining these solution methods, scalability is significantly improved, which will be demonstrated on the complete vehicle energy management problem that we will introduce in the next section.

IV. APPLICATION TO A VEHICLE POWER NET

The distributed optimization approach presented in the previous sections will be used to find the optimal solution for energy management of a vehicle with an internal combustion engine (ICE), an electric machine, a high-voltage battery, and a control system (CCS). The topology is shown in Fig. 2. This topology has three exogenous load signals, i.e., \( v_1, v_2, v_3 \in \mathbb{R}^3 \), which are the power required to drive a certain drive cycle, the power required for uncontrolled high-voltage auxiliaries and the power required for controlled low-voltage auxiliaries, respectively. These three signals are assumed to be known for every time instant \( k \in \mathbb{K} \). Furthermore, we assume that the gearsplit strategy is fixed such that the rotational velocity of the drive line \( \omega_{dc} \) is known for every time instant \( k \in \mathbb{K} \). We also assume that the power losses in the gearbox are negligible, i.e., \( y_{lbh,k} = u_{lbh,k} \), such that nodes connecting the subsystems are given by

\[
\begin{align*}
    v_{1,k} - y_{lbh,k} - y_{ic,k} + u_{em,k} + u_{alt,k} - y_{cco,k} &= 0, \\
    v_{2,k} - y_{lbh,k} - y_{lbv,k} - y_{at,k} - y_{ao,k} + u_{dc,k} &= 0, \\
    v_{3,k} - y_{lbh,k} - y_{lbv,k} - y_{br,k} &= 0,
\end{align*}
\]

where \( m = \{ice, em, hvb\} \), and \( y_{lbh,k} = u_{lbh,k} \) are the energy losses in the converter and the converter minus the energy flowing out of the converter, respectively. The energy losses in each converter are given by

\[
\begin{align*}
    c_{m} = \tau, \\
    d_{m} = \tau
\end{align*}
\]

for \( m \in \{ice, em, alt, dc, hvb, lbv\} \). For the refrigerated trailer, the air supply system, the climate control system and the mechanical brakes, all energy flowing into the subsystem is eventually lost and therefore the energy losses are given for

\[
\begin{align*}
    c_{m} &= 0, \\
    d_{m} &= \tau
\end{align*}
\]

for \( m \in \{alt, as, ccs, br\} \). By substituting (33) into (1a), we obtain

\[
\begin{align*}
    \min_{\{u_{m,k} \}_{m \in M}} & \sum_{m \in M} \sum_{k \in \mathbb{K}} c_{m} u_{m,k} - d_{m} y_{m,k} = \\
    \min_{\{u_{m,k} \}_{m \in M}} & \tau \left( u_{ic,k} - y_{ic,k} + u_{em,k} + u_{alt,k} - y_{cco,k} - y_{lbv,k} - y_{lbh,k} - y_{at,k} - y_{ao,k} + u_{dc,k} - y_{lbv,k} - y_{lbh,k} - y_{br,k} + u_{lbv,k} + u_{lbh,k} \right)
\end{align*}
\]

By using the power balance constraints (30), we can reduce this equation to

\[
\begin{align*}
    \min_{\{u_{ic,k}, u_{lbv,k}, u_{lbh,k} \}} & \sum_{k \in \mathbb{K}} \tau u_{ic,k} - \tau \left( v_{1,k} + v_{2,k} + v_{3,k} \right) + \tau (u_{lbv,k} + u_{lbh,k})
\end{align*}
\]

Moreover, as will be explained in more detail in the next subsection, the high-voltage battery and low-voltage battery satisfy integrator dynamics so that we can write \( \sum_{k \in \mathbb{K}} \tau u_{m,k} = \)
\[ x_{m,k} - x_{m,K} \text{ for } m \in \{ hvb, lvb \} \] and further reduce (34b) to
\[
\min_{\{ u_{ice,k} \}} \sum_{k \in K} \tau_{u_{ice,k}} - \tau (v_{1,k} + v_{2,k} + v_{3,k}) + x_{hvbc,0} - x_{hvbc,K} + x_{lvc,0} - x_{lvc,K}. \quad (34c)
\]
As the load signals are known for all \( k \in K \) and the initial and final states \( x_{m,0} \text{ and } x_{m,K} \) for \( m \in \{ hvb, lvb \} \) are fixed, the optimal value for \( u_{ice,k} \) for (34c) is equivalent to the optimal value \( u_{ice,k} \) for (32) for all \( k \in K \). Hence, the optimal control problem (1) with objective function (1a) and \( c_{\text{em}} \) and \( d_{\text{em}} \) given by (33) provides the optimal solution for which the fuel consumption is minimized over all \( k \in K \) as is indicated in (32).

**B. Subsystem Modeling**

The models for the internal combustion engine, the electric machine, the alternator, the DCDC converter, the mechanical brakes the high-voltage battery, the low-voltage battery, the refrigerated semi-trailer, the air supply system and the climate control system will be presented below.

1) **Internal combustion engine, electric machine and alternator:** The input-output behavior of the internal combustion engine, the electric machine and alternator can be described by the quadratic function (1b) for \( m \in \{ \text{ice, em, alt} \} \) where the efficiency coefficients depend on speed, i.e.,
\[
q_{m,k} = q_{m} (\omega_k), \quad f_{m,k} = f_{m} (\omega_k), \quad c_{m,k} = c_{m} (\omega_k), \quad (35)
\]
for \( m \in \{ \text{ice, em, alt} \} \) where \( q_{m} (\omega_k), f_{m} (\omega_k) \) and \( c_{m} (\omega_k) \) are functions parameterizing the efficiency coefficients as function of drive line speed \( \omega_k \). The input power of the converters are bounded by (1d) for \( m \in \{ \text{ic}, \text{em}, \text{alt} \} \) where the upper and lower bound depend on speed, i.e.,
\[
\omega_m,k \leq \omega_m (\omega_k), \quad \omega_m,K = \omega_m (\omega_k). \quad (36)
\]
for \( m \in \{ \text{ic}, \text{em}, \text{alt} \} \) where \( \omega_m (\omega_k) \) and \( \omega_m (\omega_k) \) are functions parameterizing the lower and upper bound as function of drive line speed \( \omega_k \). The input-output behavior is shown in Fig. 3 where the power losses, i.e., \( \omega_m,k - \omega_m,K \) for the internal combustion engine, electric machine and alternator for two different speeds. In this figure, we show measured efficiencies of typical components in a truck and the accuracy of the quadratic approximations. We normalized the power losses to avoid sharing confidential information. Still, the figure shows that the quadratic behavior, the lower and upper bound strongly depend on the drive line speed \( \omega_k \). Furthermore, the quadratic assumption on the behavior of converters holds well as the models are close to the measurement data. To be precise, the average root mean square error over all drive line speeds is 2.32 kW, 0.18 kW and 0.05 kW for the internal combustion engine, the electric machine and the alternator, respectively. The internal combustion engine, the electric machine and the alternator are subsystems without a (constrained) energy storage. Therefore, constraints on the system dynamics (1c) and state constraints (1e) do not have to be taken into account in these subsystems.

2) **DCDC converter and mechanical brakes:** The input-output behavior of the DCDC converter and mechanical brakes can be described by the quadratic function (1b) for \( m \in \{ \text{dc, br} \} \) with efficiency coefficients \( q_{m,k} \in \mathbb{R}, f_{m,k} \in \mathbb{R} \) and \( c_{m,k} \in \mathbb{R} \) for \( m \in \{ \text{dc, br} \} \) which do not depend on speed. The input power is bounded by (1d) for \( m \in \{ \text{dc, br} \} \). The DCDC converter and the mechanical brakes are also subsystems without a (constrained) energy storage. Therefore, constraints on the system dynamics (1c) and state constraints (1e) do not have to be taken into account in these subsystems.

3) **High- and low-voltage battery:** The models of the high- and low-voltage battery are derived from a battery equivalent circuit model, i.e., an open circuit voltage \( U_{m,oc} \) for \( m \in \{ hvb, lvb \} \) in series with a resistance \( R_{m,oc} \) for \( m \in \{ hvb, lvb \} \) (see, e.g., [2]), which lead to an input-output behavior of the converter that can be described by the quadratic function (1b) for \( m \in \{ hvb, lvb \} \) with \( q_{m,k} = q_{m,ocl} = \frac{R_{m,oc}}{\tau_{m,ocl}}, f_{m,k} = \frac{1}{\tau_{m,ocl}} \) and \( c_{m,k} = 0 \) for \( m \in \{ hvb, lvb \} \). The input power of the high-voltage and low-voltage battery are bounded by (1d) for \( m \in \{ hvb, lvb \} \). The dynamics are given by (1c) for \( m \in \{ hvb, lvb \} \) with \( A_{m} = 1, B_{m,w} = 0 \) and \( B_{m,a} = -\tau \) with \( \tau \) being the sample time. Here, the state \( x_{m,k} \) represents the energy in the battery at time instant \( k \).

4) **Refrigerated semi-trailer:** The input-output behavior of the converter in the refrigerated semi-trailer can be described by the quadratic function (1b) for \( m \in \{ \text{rst} \} \) and the input power of the converter is bounded by (1d) for \( m \in \{ \text{rst} \} \). The dynamics of the refrigerated semi-trailer are assumed to satisfy a thermal energy balance (see, e.g., [34]) given by
\[
C_{\text{rst}} \frac{dT_{\text{rst}}}{dt} = (u_{\text{rst}} - h(T_{\text{rst}} - \alpha T_{\text{amb}})), \quad (37)
\]
where \( C_{\text{rst}} \) is the heat capacity of the refrigerated semi-trailer and its contents, \( u_{\text{rst}} \) is the thermal power where negative
values indicate cooling, \( h \) is the heat transfer coefficient between the refrigerated semi-trailer and the environment, \( \alpha \) is an insulation coefficient, \( T_{\text{ref}} \) is the temperature inside the refrigerated semi-trailer and \( T_{\text{amb}} \), is the ambient temperature (which is assumed to be constant). Similar to the battery, we can represent the refrigerated semi-trailer model in terms of stored energy by defining the thermal energy relative to the ambient temperature \( x_{\text{ras}} = C_{\text{r}}(\alpha(T_{\text{amb}} - T_{\text{ref}})) \). By doing so and by making a forward Euler approximation of (37), the dynamics can be represented by (1c) for \( m \in \{\text{rst}\} \) with \( A_{\text{ref}} = 1 - \frac{\gamma}{c_{\text{ps}}} \), \( B_{\text{ref},w} = 0 \) and \( B_{\text{ref},u} = -\tau \) for all \( k \in \mathbb{K} \).

5) Air supply system: The input-output behavior of the converter in the air supply system can be described by the quadratic function (1b) for \( m \in \{\text{rst}\} \) and the input power of the converter is bounded by (1d) for \( m \in \{\text{ias}\} \). The dynamics of the air supply system are assumed to satisfy a mass energy balance (see, e.g., [22]) given by

\[
\frac{d}{dt} P_{\text{vas}} = R(T_{\text{in}}m_{\text{in}} - T_{\text{out}}m_{\text{out}}).
\]

where \( R \) is the specific gas constant for air, \( V \) is the lumped volume of the air vessels, \( m_{\text{in}} \) is the mass flow into the air vessels with air temperature \( T_{\text{in}} \) and \( m_{\text{out}} \) is the mass flow out of the air vessels with air temperature \( T_{\text{out}} \). Similar to the battery, we can represent the air supply system model in terms of stored energy by defining the pneumatic energy relative to the ambient pressure \( x_{\text{vas}} = \frac{R}{c_{\text{ps}}}(P_{\text{vas}} - P_{\text{amb}}) \) where \( \gamma = c_{p}/c_{\text{ps}} \) is the ratio of specific heats (approximately 1.4 for air).

Furthermore, we define the pneumatic power by \( u_{\text{vas}} = \frac{\tau_{\text{vas},w}}{c_{\text{ps}}} \). By doing so and by making a forward Euler approximation of (38), the dynamics can be represented by (1c) for \( m \in \{\text{ras}\} \) with \( A_{\text{vas}} = 1 - \frac{\tau_{\text{vas},w}}{c_{\text{ps}}} \), \( B_{\text{vas},w} = -\tau \), \( B_{\text{vas},u} = \tau \) and \( u_{\text{vas},k} \) is the latent heat \( Q_{\text{e}} \) at time instant \( k \).

6) Climate control system: The input-output behavior of the converter in the climate control system can be described by the quadratic function (1b) for \( m \in \{\text{ccs}\} \) where the efficiency coefficients are speed dependent, i.e.,

\[
\varphi_{\text{ccs},k} = \varphi_{\text{ccs}}(\omega_{k}), \quad f_{\text{ccs},k} = f_{\text{ccs}}(\omega_{k}), \quad e_{\text{ccs},k} = e_{\text{ccs}}(\omega_{k}).
\]

where \( \varphi_{\text{ccs}}(\omega_{k}) \), \( f_{\text{ccs}}(\omega_{k}) \) and \( e_{\text{ccs}}(\omega_{k}) \) are functions parameterizing the efficiency coefficients as function of the drive line speed \( \omega_{k} \). The input power of the climate control system is bounded by (1d) for \( m \in \{\text{ccs}\} \) where the bounds depend on speed, i.e.,

\[
\underline{\varphi}_{\text{ccs},k} = \underline{\varphi}_{\text{ccs}}(\omega_{k}), \quad \overline{\varphi}_{\text{ccs},k} = \overline{\varphi}_{\text{ccs}}(\omega_{k}),
\]

where \( \underline{\varphi}_{\text{ccs}}(\omega_{k}) \) and \( \overline{\varphi}_{\text{ccs}}(\omega_{k}) \) are functions parameterizing the lower and upper bound as function of drive line speed \( \omega_{k} \). The dynamics of the climate control system are assumed to satisfy a coupled thermal energy balance (see, e.g., [35]) given by

\[
C_{l} \frac{d}{dt} T_{\text{c}} = h_{l}(T_{\text{w}} - T_{\text{c}}) + Q_{\text{c}},
\]

\[
C_{w} \frac{d}{dt} T_{\text{w}} = Q_{\text{c}} + h_{w}(T_{\text{amb}} - T_{\text{w}}) + h_{l}(T_{\text{c}} - T_{\text{w}}),
\]

where \( C_{l} \) and \( C_{w} \) are the heat capacities of the refrigerant and walls of the evaporator, respectively, \( T_{\text{c}} \) and \( T_{\text{w}} \) are the temperatures of the refrigerant and walls of the evaporator, respectively, \( Q_{\text{c}} \) is the cooling power from the compressor, \( T_{\text{amb}} \) is the ambient temperature, \( h_{l} \) and \( h_{w} \) are the heat transfer coefficients between the inner and outer walls of the evaporator, respectively and \( Q_{l} \) is the latent heat. Similar to the battery, we can represent the climate control system model in terms of stored energy by defining the thermal energy in the wall and refrigerant relative to the ambient temperature, i.e., \( x_{\text{ras}} = [C_{l}(T_{\text{c}} - T_{\text{amb}}), C_{w}(T_{\text{w}} - T_{\text{amb}})]^{T} \).

By doing so and by making a forward Euler approximation of (41), the dynamics can be represented by (1c) for \( m \in \{\text{ccs}\} \) with \( A_{\text{ccs}} = 1 - \frac{\tau_{\text{ccs},w}}{c_{\text{ps}}} \), \( B_{\text{ccs},w} = \left[\begin{array}{c} 1 - \frac{\tau_{\text{ccs},w}}{c_{\text{ps}}} \\
\frac{\tau_{\text{ccs},u}}{c_{\text{ps}}} \end{array}\right] \), \( B_{\text{ccs},u} = 0 \) and \( u_{\text{ccs},k} \) is the latent heat \( Q_{\text{c}} \) at time instant \( k \).

The CVEM problem for a vehicle with an internal combustion engine, an electric machine, a high-voltage battery, a refrigerated semi-trailer, an air supply system, an alternator, a DCDC converter, a low-voltage battery and a climate control system is now fully described by the optimal control problem defined in Section II. In particular, the topology of the vehicle is described through (1f) for a given \( A_{\text{m}}, B_{\text{m}}, \) the objective function, i.e., minimizing fuel consumption, is described by (1a) by choosing \( c_{\text{m}} \) and \( d_{\text{m}} \) appropriately and the behavior of each subsystem is fully described by (1b) - (1e) by choosing the efficiency coefficients \( \varphi_{\text{m},k}, f_{\text{m},k} \) and \( e_{\text{m},k} \) the state-space matrices \( A_{\text{m}}, B_{\text{m}}, \) \( B_{\text{m},w} \) describing the dynamics of the subsystems, the upper and lower bound on the inputs \( u_{\text{m},k} \) and \( u_{\text{m},k} \), respectively, and the upper and lower bounds on the states \( x_{\text{m},k} \) and \( \tau_{\text{m},k} \), respectively. This allows the CVEM problem to be solved with the solution methods proposed in Section II and Section III as will be demonstrated in the next section.

V. SIMULATION RESULTS

In this section, we will demonstrate the distributed optimization approach to complete vehicle energy management (CVEM) by using a simulation study. First, we will give the exogenous signals that we used for the simulation study followed by the results that will be discussed in three subsections. In the first subsection, we will analyze the computational performance and comparing it with the state-of-the art solver CPLEX [30]. In the second subsection, we will discuss the optimal power flows and state trajectories and, in the last subsection, the fuel consumption reduction for CVEM will be discussed.

A. Exogenous Inputs

The driving cycle is commonly described by a velocity profile over time. If the gear shift strategy is assumed to be known, the velocity profile can be converted to a power required at the wheels and the engine speed. This set of data is derived for a PAN European driving cycle and shown in Fig. 4. It can be seen that the brake power can reach -1000 kW. However, only a small part of the total braking power can be recovered by the subsystems. Therefore, the power request used as load signal \( v_{\text{c},k} \) is limited to the maximum braking power that can
be recovered with all subsystems combined. Furthermore, the uncontrolled high-voltage auxiliaries are assumed to be absent such that $v_{2,k} = 0$ kW for all $k \in K$ and the power required from uncontrolled low-voltage auxiliaries is assumed to be constant, i.e., $v_{3,k} = 1.5$ kW for all $k \in K$. A quasi-static approach is generally sufficient for energy management (see, e.g., [3]) such that the sample time is chosen to be 1 second, i.e., $\tau = 1$, which is smaller than the time constants of the dynamics in the subsystems.

### B. Computational Performance

The local optimization problem related to each component defined in (13a) can be solved via different solution methods introduced in Section III. In particular, the performance of the ADMM solution method depends on the penalty parameter $R$ in (18) and the interval length $K_k - K_{k-1}$ for $k \in L$. For simplicity we assume that the interval length is equal for all intervals $\ell \in L$. The maximum, average and minimum time to compute the solution of the local optimization problem (13a) is given in Table I for the high-voltage battery (HVB), the refrigerated semi-trailer (RST), the air supply system (AS) and the climate control system (CCS) for different values of $K_k - K_{k-1}$ and two horizon lengths $K$. The value of the penalty parameter $R$ in (18) is manually tuned for each $K_k - K_{k-1}$ but is kept constant for different lengths $K$ of the drive cycle.

The computation time required to solve the optimization problem strongly depends on the amount of iterations $t$, as in (18), required for the ADMM method to converge, which depends on the initial guess of the dual variables. We use the dual variables from the previous iteration $s$ in the dual decomposition, see (9), as an initial guess and therefore the amount of ADMM iterations reduces as the dual decomposition converges. As a consequence, the maximum time is significantly larger than the minimum time. The main conclusion drawn from this table is that the optimal $K_k - K_{k-1}$ differs per component and it should be chosen neither too small nor too large. Moreover, it does not seem to depend on $K$. For example, the high-voltage battery has the highest performance for $K_1 - K_{0} = 200$ which is the optimal trade-off between the size and number of QPs.

With the results of Table I, we choose $K_1 - K_{0} = 200$ for the high-voltage battery, $K_1 - K_{0} = 50$ for the refrigerated semi-trailer, $K_1 - K_{0} = 200$ for the air supply system and $K_1 - K_{0} = 50$ for the climate control system. These results are compared with other solution methods in Table II. This table shows the average computation time to solve the local optimization problem (13a) over all iterations $s$ in the dual decomposition. Here, QP indicates the computation time for solving the local optimization problem (13a) with the QP solver CPLEX [30] directly, ADMM corresponds with section III.A and LM corresponds with the Lagrangian Method introduced in Section III.B. This table shows that ADMM offers a large improvement compared to QP, especially for large horizons $K$. The LM method (with relaxation parameter $\gamma = 1$) reduces the computation time even further and depends on the number of intervals $L$ required for the LM method. For $K = 3000$, the amount of intervals are 2, 6 and 53 for the high-voltage battery, the air supply system and the refrigerated semi-trailer, respectively. Note that this method cannot be used for the climate control system as this method is only suitable for scalar-state systems, i.e., $x_{m,k} \in \mathbb{R}$. Due to the absence of state constraints, the optimization problem for the internal combustion engine, electric machine, alternator, DC/DC converter and mechanical brakes can be solved explicitly and are not shown in the table. Since the dynamics of the low-voltage battery are similar to the dynamics of the high-voltage battery, we conjecture that LM is also best for the low-voltage battery.

To assess the computational performance of solving the energy management problem for different vehicle configurations, we define six case studies with increasing complexity. These case studies are introduced in Table III. To demonstrate that the conditions in Theorem 1 hold, we show in Table IV the minimum value of the dual variables $\text{min}_{m,k} (\mu_m^{(s)})$ over all iterations in the dual decomposition algorithm where (i) denotes the $i$-th element of the vector $\mu_k \in \mathbb{R}$. Note that with $B_m = -I$ for all $m \in M$ and $d_m = \tau$ for all $n \in M$, the condition in Theorem 1 is satisfied if and only if $\mu_m^{(s)} > -\tau$ for all $k \in K$ and $i \in \{1, 2, 3\}$, which is always satisfied as shown.

**TABLE I: Computation time in seconds for ADMM method with different interval lengths**

<table>
<thead>
<tr>
<th>$K$</th>
<th>1000</th>
<th>5000</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>min</td>
<td>av</td>
</tr>
<tr>
<td>HVB</td>
<td>25</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td>0.16</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>0.71</td>
</tr>
<tr>
<td>RST</td>
<td>25</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>0.17</td>
</tr>
<tr>
<td>AS</td>
<td>25</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>0.09</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td>0.18</td>
</tr>
<tr>
<td></td>
<td>500</td>
<td>0.99</td>
</tr>
<tr>
<td>CCS</td>
<td>25</td>
<td>0.1</td>
</tr>
<tr>
<td></td>
<td>50</td>
<td>0.1</td>
</tr>
<tr>
<td></td>
<td>100</td>
<td>0.12</td>
</tr>
<tr>
<td></td>
<td>200</td>
<td>0.14</td>
</tr>
</tbody>
</table>
TABLE II: Average computation time in seconds per component, per dual decomposition iteration

<table>
<thead>
<tr>
<th></th>
<th>K=1000</th>
<th>K=2000</th>
<th>K=3000</th>
<th>K=4000</th>
<th>K=5000</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>HVB</td>
<td>QP</td>
<td>2.81</td>
<td>23.0</td>
<td>93.0</td>
<td>253.8</td>
</tr>
<tr>
<td></td>
<td>ADMM</td>
<td>0.41</td>
<td>1.22</td>
<td>2.37</td>
<td>3.32</td>
</tr>
<tr>
<td></td>
<td>LM</td>
<td>2.5e-04</td>
<td>0.066</td>
<td>0.063</td>
<td>0.07</td>
</tr>
<tr>
<td>RST</td>
<td>QP</td>
<td>3.93</td>
<td>29.4</td>
<td>114.8</td>
<td>291.1</td>
</tr>
<tr>
<td></td>
<td>ADMM</td>
<td>0.72</td>
<td>1.38</td>
<td>1.94</td>
<td>2.60</td>
</tr>
<tr>
<td></td>
<td>LM</td>
<td>0.045</td>
<td>0.21</td>
<td>0.38</td>
<td>0.74</td>
</tr>
<tr>
<td>AS</td>
<td>QP</td>
<td>4.98</td>
<td>33.9</td>
<td>130.7</td>
<td>366.3</td>
</tr>
<tr>
<td></td>
<td>ADMM</td>
<td>0.44</td>
<td>0.59</td>
<td>1.15</td>
<td>3.09</td>
</tr>
<tr>
<td></td>
<td>LM</td>
<td>0.012</td>
<td>0.004</td>
<td>0.046</td>
<td>0.071</td>
</tr>
<tr>
<td>CCS</td>
<td>QP</td>
<td>3.50</td>
<td>40.2</td>
<td>89.6</td>
<td>136.9</td>
</tr>
<tr>
<td></td>
<td>ADMM</td>
<td>0.28</td>
<td>0.7</td>
<td>0.93</td>
<td>1.23</td>
</tr>
</tbody>
</table>

in Table IV for \( r = 1 \). Theorem 1 is satisfied for all simulations and is not further demonstrated in this section. Moreover, this table also shows the reduced iterations of the dual Newton update strategy compared with an update strategy with fixed step sizes, i.e., with \( \alpha_k^f \) being a constant. The Newton strategy always converged which implies that the derivatives in (12) are sufficiently well approximated.

The computation times are given in Table V for each configuration. For these simulations, the optimal control problems related to each subsystem are solved in series which is more straightforward to implement and moreover, this results already in sufficient computational benefits for offline energy management. The computation time of the Distributed Optimization (DO) method introduced in this paper are compared with the computation time of the QCQP solver CPLEX [30]. The CPLEX solver cannot handle quadratic constraints written in vector format and every quadratic constraint needs to be programmed separately. This requires a large amount of assembly time which is not used for solving the actual optimization problem. Therefore, the computation times of CPLEX with and without assembling the optimization problem are given. If we compare only the time required to solve the optimization problem, DO is still 1825 times faster for Case 1 with \( K = 5000 \) and 64 times faster for Case 6 with \( K = 3000 \). Scalability of DO in the horizon length \( K \) is superior compared with CPLEX. Scalability in the number of components is not always better with DO but only in the rare case with small \( K \) and many more components, CPLEX could be better than DO. The flexibility of adding and removing components with CPLEX remains poor though.

TABLE III: Case studies with problem size defined in number of inputs, states and quadratic constraints.

<table>
<thead>
<tr>
<th>Case</th>
<th>Inputs</th>
<th>States</th>
<th>Quad. Constr.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Truck with ICE, EM and a HVB</td>
<td>4K</td>
<td>K</td>
</tr>
<tr>
<td>2</td>
<td>Case 1 with a RST</td>
<td>3K</td>
<td>K</td>
</tr>
<tr>
<td>3</td>
<td>Case 2 with an AS</td>
<td>6K</td>
<td>3K</td>
</tr>
<tr>
<td>4</td>
<td>Case 3 with a CCS</td>
<td>7K</td>
<td>5K</td>
</tr>
<tr>
<td>5</td>
<td>Case 4 with an ALT and a LVB</td>
<td>9K</td>
<td>6K</td>
</tr>
<tr>
<td>6</td>
<td>Case 5 with a DCDC converter</td>
<td>10K</td>
<td>6K</td>
</tr>
</tbody>
</table>

TABLE IV: Number of iterations and minimum of the dual variables over all iterations for \( K = 5000 \)

<table>
<thead>
<tr>
<th></th>
<th>( \text{iter} ) (( \alpha_k^f ) constant)</th>
<th>( \text{iter} ) (Newton)</th>
<th>( \min \lambda_k^E )</th>
<th>( \min \lambda_k^L )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>105</td>
<td>122</td>
<td>119</td>
<td>228</td>
</tr>
<tr>
<td>Case 2</td>
<td>26</td>
<td>30</td>
<td>31</td>
<td>113</td>
</tr>
<tr>
<td>Case 3</td>
<td>0.416</td>
<td>0.450</td>
<td>0.431</td>
<td>-0.03</td>
</tr>
<tr>
<td>Case 4</td>
<td>0.623</td>
<td>0.622</td>
<td>0.626</td>
<td>0.016</td>
</tr>
<tr>
<td>Case 5</td>
<td>N.A.</td>
<td>N.A.</td>
<td>N.A.</td>
<td>0.322</td>
</tr>
</tbody>
</table>

TABLE V: Computation times of DO and CPLEX in seconds

<table>
<thead>
<tr>
<th></th>
<th>K=1000</th>
<th>K=2000</th>
<th>K=3000</th>
<th>K=4000</th>
<th>K=5000</th>
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<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case 1</td>
<td>CPLEX(^1)</td>
<td>8.1</td>
<td>1.28</td>
<td>80</td>
<td>300</td>
</tr>
<tr>
<td></td>
<td>CPLEX(^2)</td>
<td>46</td>
<td>1405</td>
<td>4033</td>
<td>6509</td>
</tr>
<tr>
<td></td>
<td>DO</td>
<td>0.035</td>
<td>0.24</td>
<td>0.21</td>
<td>0.74</td>
</tr>
<tr>
<td>Case 2</td>
<td>CPLEX(^1)</td>
<td>20.3</td>
<td>369</td>
<td>1126</td>
<td>2335</td>
</tr>
<tr>
<td></td>
<td>CPLEX(^2)</td>
<td>90</td>
<td>942</td>
<td>3085</td>
<td>6946</td>
</tr>
<tr>
<td></td>
<td>DO</td>
<td>0.14</td>
<td>0.99</td>
<td>1.19</td>
<td>1.96</td>
</tr>
<tr>
<td>Case 3</td>
<td>CPLEX(^1)</td>
<td>45</td>
<td>64</td>
<td>2014</td>
<td>6650</td>
</tr>
<tr>
<td></td>
<td>CPLEX(^2)</td>
<td>146</td>
<td>1177</td>
<td>4863</td>
<td>13528</td>
</tr>
<tr>
<td></td>
<td>DO</td>
<td>0.34</td>
<td>1.72</td>
<td>1.69</td>
<td>2.74</td>
</tr>
<tr>
<td>Case 4</td>
<td>CPLEX(^1)</td>
<td>71</td>
<td>454</td>
<td>3112</td>
<td>&gt; 10(^8)</td>
</tr>
<tr>
<td></td>
<td>CPLEX(^2)</td>
<td>357</td>
<td>2723</td>
<td>10770</td>
<td>&gt; 10(^8)</td>
</tr>
<tr>
<td></td>
<td>DO</td>
<td>4.35</td>
<td>10.5</td>
<td>18.62</td>
<td>24.82</td>
</tr>
<tr>
<td>Case 5</td>
<td>CPLEX(^1)</td>
<td>76</td>
<td>938</td>
<td>2834</td>
<td>&gt; 10(^8)</td>
</tr>
<tr>
<td></td>
<td>CPLEX(^2)</td>
<td>791</td>
<td>6835</td>
<td>22213</td>
<td>&gt; 10(^8)</td>
</tr>
<tr>
<td></td>
<td>DO</td>
<td>9.6</td>
<td>23.1</td>
<td>43.8</td>
<td>69.9</td>
</tr>
<tr>
<td>Case 6</td>
<td>CPLEX(^1)</td>
<td>79</td>
<td>965</td>
<td>3285</td>
<td>&gt; 10(^8)</td>
</tr>
<tr>
<td></td>
<td>CPLEX(^2)</td>
<td>1003</td>
<td>8304</td>
<td>28202</td>
<td>&gt; 10(^8)</td>
</tr>
<tr>
<td></td>
<td>DO</td>
<td>11.3</td>
<td>31.8</td>
<td>51.4</td>
<td>73.2</td>
</tr>
</tbody>
</table>

C. Optimal Input and State Trajectories

The optimal power flows as function of time are shown in Fig. 5 for the complete vehicle and with a drive cycle length of only \( K = 3000 \) for clarity. Both, the results from DO, as well as the results from solving the optimization problem with CPLEX are shown. This figure demonstrates that both methods converge to the same solution (within a desired tolerance). Moreover, the fuel consumption of DO is 0.019 % smaller compared with CPLEX, which is negligible. Two important observations can be made from this figure, 1) all auxiliaries are used to store (brake) energy and 2) the DCDC converter is generally used to supply the low-voltage auxiliaries, except when free brake energy is available, then the alternator supplies the low-voltage auxiliaries and charges the low-voltage battery. Observation 1 can also be seen from the state trajectories given in Fig. 6 where \( \bar{x}_{hub} = \frac{x_{hub}}{x_{hub,0}} \) is the high-voltage battery energy normalized with respect to the maximum battery capacity \( E_{hub,0} \), \( E_{hub} \) is the low-voltage battery energy normalized with respect to the maximum battery capacity \( E_{hub,0} \), \( T_{hub} \) is the air temperature in the refrigerated trailer, \( P_{wa} \) is the air pressure in the air supply system and \( T_{wa} = [T_{wa} \; T_{wa}^T] \) is the wall and refrigerant temperature in the climate control system. This figure shows that all state constraints are met, where for the climate control system, only the constraint on the wall temperature is shown.
D. Fuel Consumption Reduction

To analyze the fuel consumption for different parts of the complete drive cycle, the drive cycle is split into three parts. The first part is given by $k \in \{0, \ldots, 19999\}$, the second part by $k \in \{20000, \ldots, 39999\}$ and the third part by $k \in \{40000, \ldots, 55579\}$. The fuel consumption reduction for each of the cases and for each of these drive cycles are given in Table VI. For the first case, the baseline is a non-hybrid truck with the air temperature in the refrigerated semi-trailer kept at its upper bound. The fuel reduction for some auxiliaries, e.g., the air supply system kept at its upper bound and the temperature in the climate control system kept at its upper bound. For the next cases, the baseline is the previous case to emphasize the potential of each auxiliary. The DCDC converter (Case 6) is the most potential auxiliary for reducing fuel with 0.34%.

Although, the current trend in automotive applications is to electrify the auxiliaries in the vehicle which do allow for continuous control (see, e.g., [36]), still the refrigerated semi-trailer, the air supply system and the climate control system are often attached to the engine via a clutch and, as such, cannot be continuous controlled. These auxiliaries are switched between an on and off state. Therefore, comparing with a baseline continuous controller gives not the full potential of CVEM. The last column in Table VI is therefore added which uses a baseline controller where the auxiliary is turned on when the state hits the lower bound and turned off when the state hits the upper bound. The fuel reduction for the refrigerated semi-trailer and climate control system with the switched baseline is much higher, i.e., 0.42% and 0.21%, respectively. However, this requires a switched controller with many more switches compared with the baseline. More switches will reduce the life time of the auxiliaries and an optimal trade-off must be found between the number of switches and the fuel reduction.

The fuel reduction for some auxiliaries, e.g., the air supply system and the climate control system, is very low and integration of those auxiliaries into the energy management strategy might not outweigh the additional cost and complexity. However, the main result of the distributed optimization approach presented in this paper is that the energy management problem is decomposed into smaller energy management problems related to each subsystem. Each of the energy management problems on subsystem level is much easier to solve and can be solved with different algorithms, e.g., an ADMM method or a Lagrangian Method. Extensions to more sophisticated models, e.g., by including battery aging or battery thermal dynamics, will be easier and part of future work. Moreover, developing optimal control algorithms for different subsystems can be done in parallel, e.g., thermal management of the internal combustion engine can be included in the internal combustion engine optimization problem, while at the same time thermal management of the auxiliaries can be done in parallel. However, this requires a switched controller with many more switches compared with the baseline. More switches will reduce the life time of the auxiliaries and an optimal trade-off must be found between the number of switches and the fuel reduction.

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![Fig. 5: Optimal power flows (in kW) for DO and CPLEX](image)

![Fig. 6: Optimal state trajectories for DO.](image)

### TABLE VI: Fuel consumption reduction results.

<table>
<thead>
<tr>
<th>Case</th>
<th>Part 1</th>
<th>Part 2</th>
<th>Part 3 Complete</th>
<th>Part 1 (switched)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.06%</td>
<td>-10.9%</td>
<td>-2.35%</td>
<td>-0.64%</td>
</tr>
<tr>
<td>2</td>
<td>-0.08%</td>
<td>-0.12%</td>
<td>-0.03%</td>
<td>-0.07%</td>
</tr>
<tr>
<td>3</td>
<td>-0.02%</td>
<td>-0.03%</td>
<td>-0.01%</td>
<td>-0.02%</td>
</tr>
<tr>
<td>4</td>
<td>-0.01%</td>
<td>-0.04%</td>
<td>-0.02%</td>
<td>-0.03%</td>
</tr>
<tr>
<td>5</td>
<td>-0.07%</td>
<td>-0.11%</td>
<td>-0.02%</td>
<td>-0.06%</td>
</tr>
<tr>
<td>6</td>
<td>-0.34%</td>
<td>-0.32%</td>
<td>-0.23%</td>
<td>-0.34%</td>
</tr>
</tbody>
</table>
management of the high-voltage battery can be included in the high-voltage battery optimization problem.

VI. CONCLUSIONS

In this paper, a distributed optimization approach has been proposed to solve the complete vehicle energy management problem of a hybrid truck with auxiliaries. A dual decomposition is applied first to the optimal control problem such that the problem related to each subsystem can be solved separately. Then, either an ADMM method or a Lagrangian method has been used to efficiently solve the optimal control problem for every subsystem in the vehicle. The proposed approach has been demonstrated by solving the complete vehicle energy management problem of a hybrid truck with a refrigerated semi-trailer, an air supply system, an alternator, a DC/DC converter, a low-voltage battery and a climate control system. Simulation results have shown that the computation time is reduced by a factor of 64 up to 1825, compared to solving the problem with the CPLEX solver, depending on the vehicle configuration and driving conditions. The fuel consumption can be reduced up to 0.52% by including auxiliaries in the energy management problem, assuming that the auxiliaries are continuous controlled. Fuel consumption can be further reduced up to 1.08% when compared to a baseline with on/off control, but the amount of switches need to increase significantly. An interesting extension amounts to finding the optimal trade-off between the maximum allowable number of switches and fuel reduction. Moreover, the presented approach can be modified to be used for real-time control, i.e., the optimal control problem can be defined over a shorter horizon. After dual decomposition, this results in small Linearly Constrained Quadratic Programs for each of the subsystems which can be solved in real-time.

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Effects of Battery Charge Acceptance and Battery Aging in Complete Vehicle Energy Management *

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Abstract: In this paper, we propose a solution to the complete vehicle energy management problem with battery charge acceptance limitations and battery aging limitations. The problem is solved using distributed optimization for a case study of a hybrid heavy-duty vehicle, equipped with a refrigerated semi-trailer for two different battery models. The first battery model takes charge acceptance into account by adding an additional energy state to the optimization problem. The second battery model includes battery aging for which a novel iterative algorithm is proposed. Simulation results show that charge acceptance limitations only have a minor effect on the solution to the energy management problem, while battery aging leads to a trade-off between battery capacity loss and fuel consumption reduction. In particular, a decrease in capacity loss by 260%, leads to a drop in fuel consumption reduction from 9.40% (without battery aging) to 8.61% (with battery aging), both when compared to a conventional vehicle. This is caused by the fact that aging limitations cause less energy to be stored in the high-voltage battery. Still, because energy can also be stored in the refrigerated semi-trailer, smart control of this refrigerated semi-trailer leads to an additional fuel reduction 0.53% in the case where battery aging is incorporated, while it was only 0.09% when battery aging was not considered. In other words, the drop in the fuel consumption reduction caused by battery aging constraints can be partly compensated by smart control of other energy buffers, which shows the true benefit of complete vehicle energy management.

Keywords: Energy Management, Smart Auxiliaries, Charge Acceptance, Battery Aging

1. INTRODUCTION

Modern vehicles are subjected to strict emission regulations, forcing car manufacturers to find ways to reduce emissions and fuel consumption. The fuel consumption of hybrid electric vehicles is reduced by adding an electric motor and a high-voltage battery to the power train and optimally controlling the energy flow between the electric motor and combustion engine. However, auxiliaries, especially in heavy-duty vehicles, add to the fuel consumption as well. Examples of such auxiliaries are an air supply system, a refrigerated semi-trailer and a climate control system. These auxiliaries provide an opportunity to be used as a buffer to temporarily store excess energy in situations where it is not optimal to store energy in the high-voltage battery system. Therefore, it is important to consider these additional energy flows in the global energy management problem, which is referred to as complete vehicle energy management (CVEM), (Kessels et al., 2012).

Typically, the global optimal solution to the energy management problem is found by Dynamic Programming (Pérez et al., 2006). This method however, suffers from an exponentially increasing computation time with the number of states. Optimization methods based on Pontryagin's Minimum Principle (Van Keulen et al., 2014) can handle computational complexity of multi-state energy management problems, e.g., battery state-of-health is included in (Ebbesen et al., 2012) and battery thermal management in (Pham et al., 2013). The solution is found by solving a two-point boundary value problem, which can be difficult in the presence of state constraints.

For this reason, solutions that use static optimization for energy management start to appear. As CVEM typically leads to a large-scale optimization problem, a distributed optimization approach to CVEM is proposed in (Romijn et al., 2014, 2015). This allows handling a large number of components and a large control horizon in the CVEM problem and results in the global optimal solution under the condition that the CVEM problem is strictly convex. The battery model used in (Romijn et al., 2014, 2015), however, is an equivalent circuit model without the dynamics of charge acceptance and restrictions on battery aging. These models are typically used in optimal control problems due to its simplicity. However, the capability of storing energy in the high-voltage battery system is inherently overestimated, which might lead to a conservative usage of the energy buffers in the auxiliaries.

Charge acceptance, i.e., the resistance of the battery against storing energy for a longer period, has, to our best knowledge, never been included in the energy management problem. Battery aging or state-of-health has

* This work has received financial support from the Horizon 2020 programme of the European Union under the grant ‘Electric Vehicle Enhanced Range, Lifetime And Safety Through INGenious battery management’ (EVERLASTING-713771).
been included in the energy management problem (see, e.g., (Ebbesen et al., 2012), (Serrao et al., 2011), (Tang and Rizzoni, 2016)) and introduces a trade-off between battery life and fuel consumption. The aforementioned papers however do not consider auxiliaries in the energy management problem.

Therefore, we will extend the CVEM problem of (Romijn et al., 2014) with the dynamics of charge acceptance and battery aging. The case study consists of a hybrid-electric heavy-duty vehicle with a refrigerated semi-trailer. The dynamics of charge acceptance can easily be integrated into the approach of (Romijn et al., 2014) as it only adds an additional state to the CVEM problem. Battery aging is included using a convex approximation of a semi-empirical battery aging model proposed in (Wang et al., 2011). This requires a nonlinear constraint to be added to the CVEM problem which is solved by iteratively solving a quadratically constrained quadratic program.

2. COMPLETE VEHICLE ENERGY MANAGEMENT

In this paper, we consider the case study of (Romijn et al., 2014), consisting of a heavy-duty vehicle that includes an internal combustion engine (ICE), an electric machine (EM), a high-voltage battery (HVB) and a refrigerated semi-trailer (RST). The topology is schematically shown in Fig. 1, in which $P_{ICE}$ and $P_{HVB}$ denote the ICE’s fuel and mechanical power, respectively, $P_{EM}$ and $P_{RST}$ the EM’s electrical and mechanical power, respectively, $P_{br}$ and $P_{bvb}$ the battery’s electrical and stored chemical power, respectively, $P_{st}$ and $P_{el}$ the refrigerated semi-trailer’s electrical and thermal power, respectively. $P_{m}$ is the mechanical braking power, $P_{ice}$ is the power required for driving, $E_{em}$ denotes the battery state of energy and $E_{et}$ denotes the thermal energy in the refrigerated semi-trailer. No power losses are assumed in the gearbox (GB) which implies that $P_{b} = P_{bvb}$. In this section, we will present the associated optimal control problem for minimizing the total fuel consumption based on the work presented in (Romijn et al., 2014).

2.1 Optimal Control Problem

The optimal control problem of CVEM is given by

$$
\min_{\{P_{m1}, P_{m2}\}, \{m \in M\}, k \in K} \sum_{m \in M} c_m P_{m1}, k + d_m P_{m2}, k
$$

(1a)

where $P_{m1}, k \in \mathbb{R}$ and $P_{m2}, k \in \mathbb{R}$ are the (scalar) inputs and outputs of the converter in subsystem $m \in M = \{ ice, em, hvb, rst \}$ at time instant $k \in K = \{0, 1, \ldots, K - 1\}$, with $K$ the horizon length. In (1a) we use the notation $\{P_{m1}, k, P_{m2}, k\}$ to indicate $\{P_{m1}, P_{m2}\} \in M$, $\forall k \in K$. This notation will be used throughout the paper for minimizing over a set. The coefficients $c_m$ and $d_m$ in (1a) can be chosen such that (1a) minimizes the energy losses in the system which is equivalent to minimizing fuel consumption (Romijn et al., 2014). The optimization problem (1a) is to be solved subject to a quadratic equality constraint describing the input-output behavior of each converter, i.e.,

$$
P_{m2}, k = \frac{1}{2} \eta_{m,k} P_{m1}^2, k + f_{m,k} P_{m1}, k + o_{m,k} E_{m,k} P_{m1}, k + e_{m,k},
$$

(1b)

for all $k \in K$, $m \in M$, and subject to system dynamics of the energy storage device in subsystem $m \in M$, i.e.,

$$
E_{m,K+1} = A_m E_{m,k} + B_m P_{m1}, k,
$$

(1c)

for all $k \in K$ where the initial state $E_{m,0}$ and final state $E_{m,K}$ of the energy storage device are assumed to be given, and the input $P_{m1}, k$ is subject to linear inequality constraints, i.e.,

$$
P_{m1}, k \leq P_{m1}, k \leq T_{m1}, k,
$$

(1d)

for all $k \in K$ and $m \in M$. Finally, the optimization problem is solved subject to linear equality constraints describing the interconnection of the subsystems, i.e.,

$$
P_{r,k} - P_{w2,k} - P_{w2,k} - P_{w2,k} = 0,
$$

(1f)

$$
P_{m1}, k - P_{m1}, k - P_{r1,k} = 0,
$$

(1g)

and

$$
C(P_{r,k}) + \sum_{m \in M} A_m P_{m1}, k + B_m P_{m2}, k = 0,
$$

(1h)

for all $k \in K$, where $A_m \in \mathbb{R}^2$, $B_m \in \mathbb{R}^2$ and $C = [1 0]^T$. This topology has one exogenous disturbance signal, i.e., the power required to drive a certain drive cycle $P_{r,k}$ which is assumed to be known for every time instant $k \in K$.

In this paper, we extend the optimal control problem with an additional constraint representing battery aging, i.e.,

$$
\sum_{k \in K} \alpha_k (P_{bvb1}, k) P_{bvb1}, k Q_{kz}^{-1} \leq Q_k - z_k,
$$

(1i)

where $\alpha_k$ is a HVB power dependent aging factor, $0 < z \leq 1$ is a battery constant, $P_{bvb1}^*$ is the maximum battery aging allowed at the end of a drive cycle with length $K$ and $Q_k$ is the battery aging at time instant $k$ given by

$$
Q_{k+z} = Q_k + \alpha_k (P_{bvb1}, k) P_{bvb1}, k Q_k^{-1}
$$

(1j)

Finally, we define the primal optimal solution as the solution $\{P_{m1}, k, P_{m2}, k\}$ that satisfies (1) and $p^*$ as the primal optimal value of (1).

2.2 Dual Decomposition

Problem (1a) subject to (1b) - (1h) cannot be separated due to the complicating constraint (1h). Therefore, we decompose the problem via dual decomposition by introducing the following so-called partial Lagrangian

$$
L(\{P_{m1}, k, P_{m2}, k, \mu_k\}) = \sum_{k \in K} C(P_{r,k} - P_{w2,k}) + \sum_{m \in M} c_m P_{m1}, k
$$

$$
+ d_m P_{m2}, k + \mu_k^2 (A_m P_{m1}, k + B_m P_{m2}, k),
$$

(2)

where $\mu_k \in \mathbb{R}^N$ is a Lagrange multiplier, subject to (1b)-(1e). Indeed, the partial Lagrangian is obtained by adding
the complicating constraints (the constraints that act on more than one subsystem) to the objective function in (1a). The partial Lagrange dual function is now given by
\[ g(\mu_k) = \min_{\{P_{m,1}, P_{m,2}\}} L(\{P_{m,1}, P_{m,2}, \mu_k\}) \]
\[ = \mu_k^T C(P_{e,k} - P_{ir,k}) + \sum_{m \in \mathcal{M}} g_m(\mu_k), \quad (3a) \]
with
\[ g_m(\mu_k) = \min_{\{P_{m,1}, P_{m,2}\}} \sum_{k \in K} c_m P_{m,1,k} + d_m P_{m,2,k} + \mu_k^T (A_m P_{m,1,k} + B_m P_{m,2,k}), \quad (3b) \]
subject to (1b)-(1j) but not subject to (1h). In (Romijn et al., 2014), it is shown that minimizing over the mechanical braking power \( P_{m,k} \) is not necessary as it is given by (1f) for the first element of \( \mu_k \), which is smaller than zero. Note that each of the Lagrange dual functions (3b) subject to (1b)-(1j), but not subject to (1h) is related to one of the subsystems and can be solved independently. The dual problem is given by
\[ \max g(\mu_k) = d^*, \quad (4) \]
subject to (1b)-(1j) except (1h) where \( d^* \) is defined as the dual optimal value. It has been shown in (Romijn et al., 2016) that under very mild and verifiable conditions, the primal and dual solution are the same, i.e., \( d^* = p^* \).

Maximizing the Lagrange dual function (3) over \( \mu_k \) can be done with a steepest ascent method. This results in an algorithm for solving the CVEM problem, given by
\[ \{P_{m,1}', P_{m,2}'\} = \arg \min_{\{P_{m,1}, P_{m,2}\}} L(\{P_{m,1}, P_{m,2}, \mu_k\}) \quad (5a) \]
\[ \mu_k^{s+1} = \mu_k^s + S_k \left( \sum_{m \in \mathcal{M}} A_m P_{m,1} + B_m P_{m,2} + C(P_{e,k} - P_{ir,k}) \right), \quad (5b) \]
subject to (1b)-(1j) but not subject to (1h) for all \( k \in K \), where \( S_k \) is a suitably chosen matrix and \( s \in \mathbb{N} \) is the iteration counter.

### 2.3 Dual Functions

Each of the Lagrange dual functions (3b) related to one of the subsystems can be solved separately and can be written as a quadratic program by substituting (1b) into (3b), which gives
\[ g_m(\mu_k) = \min_{\{P_{m,1}, P_{m,2}\}} \sum_{k \in K} \left[ \frac{1}{2} P_{m,1,k}^T H_{m,k} P_{m,1,k} + \frac{1}{2} P_{m,2,k}^T F_{m,k} P_{m,2,k} + \frac{1}{2} \gamma_m P_{m,1,k} + \frac{1}{2} \gamma_m P_{m,2,k} + C(P_{e,k} - P_{ir,k}) \right], \quad (6a) \]
with
\[ H_{m,k} = (d_m + B_m \mu_k) h_{m,k}, \quad (6b) \]
\[ F_{m,k} = c_m + A_m \mu_k + (d_m + B_m \mu_k) F_{m,B}, \quad (6c) \]
\[ G_{m,k} = (d_m + B_m \mu_k) g_{m,k}, \quad (6d) \]
subject to (1c) - (1e) and for the HVB subject to (1i) and (1j) as well. Note that for a nonzero matrix \( a_{m,k} \) in (6c), the quadratic program becomes more difficult to solve due to the cross products between \( P_{m,1,k} \) for \( k \in K \). Still, efficient solution methods for these optimization problems without (11) and (1j), i.e., an ADMM solution method and a Lagrangian solution method, are given in (Romijn et al., 2015). The battery aging constraint (1i), however, is a nonlinear constraint for which the methods proposed in (Romijn et al., 2015) cannot be used. In this paper, we will propose a modified Lagrangian solution method that takes into account the nonlinear constraint for the case that \( a_{m,k} = 0 \). In Section 4, we will show that if the dynamics of charge acceptance are not taken into account, \( a_{m,k} \) is zero. Hence, the battery optimization problem cannot be solved with this method for the combination of charge acceptance and battery aging. In Section 4, we will show that this is not a significant restriction as battery aging is dominant over charge acceptance.

### 2.4 Lagrangian Solution Method

The battery optimization problem (6) for \( m \in \{hvb\} \) subject to the nonlinear constraint (1i) is difficult to solve due to the nonlinear constraint. However, by fixing the battery aging parameters \( \alpha_k \) for all \( k \in K \) and battery aging values \( Q_k \) for all \( k \in K \), the nonlinear constraint (1i) is quadratic for which we can solve the optimization problem via a modified Lagrangian Method of (Romijn et al., 2014). This fixation basically implies that we have a (not optimal) solution \( P_{hvb,k} \) for all \( k \in K \) for which we can calculate \( \alpha_k \) and \( Q_k \) for all \( k \in K \) as in (11). By iteratively updating \( \alpha_k \) and \( Q_k \) for all \( k \in K \) in (11) with the solution of the optimization problem, the solution of the optimization problem converges to the global optimum solution.

To show this, let us define first the state-unconstrained problem for the HVB, which is given by (6) subject to (1d) and (1c) with given initial state \( E_{hvb,0} \) and final state \( E_{hvb,K} \). The Lagrangian of this problem is given by
\[ \bar{L}(\{P_{hvb,k}, \lambda_1, \lambda_2, \sigma_1, \sigma_2\}) = \left( \sum_{k \in K} \left[ \begin{array}{c} P_{hvb,k} \\ F_{hvb,k} \\ 2G_{hvb,k} \end{array} \right] \right)^T \left( \begin{array}{c} H_{hvb,k} \\ F_{hvb,k}^T \\ 2G_{hvb,k}^T \end{array} \right) \left( \begin{array}{c} P_{hvb,k} \\ F_{hvb,k} \\ 2G_{hvb,k} \end{array} \right) + \sigma_1 \left( P_{hvb,k} - \bar{P}_{hvb,k} \right) + \sigma_2 \left( F_{hvb,k} - \bar{F}_{hvb,k} \right) \right) + \frac{\lambda_1^T}{2} \left( A_{hvb,k}^T E_{hvb,0} - E_{hvb,k} + \sum_{k \in K} A_{hvb,k}^T E_{hvb,k} - E_{hvb,k} \right) + \frac{\lambda_2^T}{2} \left( B_{hvb,k}^T E_{hvb,0} - E_{hvb,k} + \sum_{k \in K} B_{hvb,k}^T E_{hvb,k} - E_{hvb,k} \right) \]
with \( \lambda_1, \lambda_2 \in \mathbb{R}^d \) the Lagrange multiplier associated with the final state constraint \( E_{hvb,K} = A_{hvb,K}^T E_{hvb,0} + \sum_{k \in K} A_{hvb,k}^T E_{hvb,k} - E_{hvb,k} \) and \( \lambda_2 \in \mathbb{R} \) the Lagrange multiplier associated with the battery aging constraint (1i). Furthermore, we have introduced \( \gamma_k = 2 \sigma_1 (P_{hvb,k}) Q_k^{\frac{1}{2}} \) which is fixed for a given \( P_{hvb,0} \) for all \( k \in K \).

The Karush-Kuhn-Tucker conditions (Boyd and Vandenberghe, 2004) for minimizing the Lagrangian in (7) are given by the first-order necessary optimality condition
\[ \partial \bar{L}(\{P_{hvb,k}, \lambda_1, \lambda_2, \sigma_1, \sigma_2\})/\partial P_{hvb,k} = H_{hvb,k} P_{hvb,k} + F_{hvb,k} + \sigma_1 - \sigma_2 + \lambda_1 B_{hvb,k} = 0, \quad (9) \]
for all \( k \in K \) with \( H_{hvb,k} = H_{hvb,k} + \lambda_1 \gamma_k \) and \( B_{hvb,k} = \sum_{k \in K} A_{hvb,k}^T E_{hvb,k} \), feasibility of the constraints (8) and...
(11) and the complementary slackness conditions for the inequality constraints
\[ \nu_k(P_{\text{hvb},k} - F_{\text{hvb},k}) = 0, \]
\[ \nu_k(P_{\text{hvb},k} - F_{\text{hvb},k}) = 0, \]
\[ \lambda_2(Q_0 - Q_t + \sum_{k \in K} \frac{1}{2} \gamma_k P_{\text{hvb},k}^2 = 0 \]
for all $k \in K$ with $\nu_k \geq 0$, $\lambda_k \geq 0$ and $\lambda_2 \geq 0$. The optimal solution to (9) and (10) is given by
\[ P_{\text{hvb},k} = -H_{\text{hvb},k}^{-1}(F_{\text{hvb},k} + \lambda_k B_{\text{hvb}} + \nu_k - \nu_k), \]
for all $k \in K$ for a given $\lambda_1$, $\lambda_2$, $\nu$ and $\nu_k$. To find the optimal solution, we propose an algorithm for which we first fix $\lambda_2$ and $\nu_k$ and solve
\[ \lambda_1^{i+1} = A_{\text{hvb},k}^T \nu_k - \frac{A_{\text{hvb},k}^T}{2} \nu_k \]
with $H_{\text{hvb}} = \sum_{k \in K} A_{\text{hvb},k}^{-1} A_{\text{hvb},k}^{-1} A_{\text{hvb},k}^{-1}$ and $\nu_k^{i+1} = \max \{0, -H_{\text{hvb},k}(F_{\text{hvb},k} - F_{\text{hvb},k} - \lambda_1^{i+1} B_{\text{hvb}}) \}$
\[ \nu_k^{i+1} = \max \{0, -H_{\text{hvb},k}(F_{\text{hvb},k} + F_{\text{hvb},k} + \lambda_1^{i+1} B_{\text{hvb}}) \} \]
with $t \in \mathbb{N}$ the iteration index and for $\nu_k^{i+1} = 0$ for all $k \in K$, until (8) is satisfied within some desired tolerance. The expressions in (12) are obtained by substituting (11) into (8) and (10). If $H_{\text{hvb},k}$ is strictly positive definite for all $k \in K$ and if there exists an optimal solution $P_{\text{hvb},k}$ for which (10) is satisfied, then the solution of (12) will converge to the solution of (6) subject to (10) and (8) but not (11).

To satisfy (11), $\gamma$ is updated after convergence of the algorithm in (12) with the optimal solution (11) by
\[ \gamma_k^{i+1} = (1 - \rho_1) \gamma_k^1 + \rho_1 (2 \nu_k (P_{\text{hvb},k}^2) - \frac{\gamma_k}{s}) \]
with $s \in \mathbb{N}$ the iteration index, $\rho_1 \in (0, 1)$ is a relaxation parameter and until $\gamma_k^{i+1} \approx \gamma_k^1$ for all $k \in K$. This algorithm will converge under the conditions that $\rho_1$ is sufficiently small and if $\gamma_k$ is a convex function of the battery power $P_{\text{hvb},k}$. Still, with $\gamma_k^{i+1} \approx \gamma_k^1$, equation (11) is not satisfied as $\lambda_2$ needs to be converged to satisfy (11). We propose to update $\lambda_2$ every time $\gamma_k$ is converged by using a ‘steepest ascent’ direction, i.e.,
\[ \lambda_2^{i+1} = \lambda_2^1 + \rho_2 (Q_0 + \sum_{k \in K} \frac{1}{2} \gamma_k P_{\text{hvb},k}^2 - Q_t) \]
with $\tau \in \mathbb{N}$ the iteration index and where $\rho_2 > 0$ is the step size.

The above presented procedure still only provides a solution for the state-unconstrained problem. For solving the state-constrained dual function, we use a property first introduced in (Van Keulen et al., 2014). In particular, we make use of the monotonic relation between the Lagrange multiplier $\lambda_1$ and the final state $E_{\text{hvb},K}$. If this relation holds, the time instant at which the unconstrained state exceeds its constraints the most, is a contact point of the state-constrained solution, and, therefore, fixes a part of the optimal solution to the optimization problem. As a result, the optimization problem can be split into two new unconstrained optimal control problems with known initial and terminal states. This property is used in (Van Keulen et al., 2014) to calculate the optimal state constrained solution by using Pontryagin’s Minimum Principle. Here, we used the same property to calculate the optimal state-constrained solution in combination with the Lagrangian method. For details, we refer to (Van Keulen et al., 2014).

3. MODELING OF COMPONENTS

After presenting the optimal control problem related to CVEM in the previous section, we now explain the models used in the optimal control problem. The models for the ICE, EM, RST will only be briefly discussed. The models for the HVB will be discussed more extensively, and extended compared to previous work in (Romijin et al., 2014). In particular, we extend in this paper the HVB model to incorporate charge acceptance and battery aging.

3.1 ICE, EM and RST

The input-output behavior of the ICE and EM can be described by the quadratic function (1b) for $m \in \{\text{ice, em}\}$, where the efficiency coefficients for the ICE and the EM are speed dependent, i.e.,
\[ q_m = q_m(\omega_m), \]
\[ f_{m,k} = f_m(\omega_m), \]
\[ e_{m,k} = e_m(\omega_m), \]
for $m \in \{\text{ice, em}\}$ where $q_m(\omega_m)$, $f_m(\omega_m)$ and $e_m(\omega_m)$ are functions parameterizing the efficiency coefficients as function of drive line speed $\omega$. The input power of the ICE and EM are bounded by (1d) for $m \in \{\text{ice, em}\}$ where the upper and lower bound for the ICE and the EM are speed dependent, i.e.,
\[ P_{m,k} = \begin{cases} p_m(\omega_k), & m \in \{\text{ice, em}\} \end{cases} \]
\[ P_{m,k} = P_{m,k}(\omega_k), \]
are functions parameterizing the lower and upper bound as function of drive line speed $\omega$. Constraints on the system dynamics (1c) and state constraints (1e) do not have to be taken into account as the ICE and EM do not have a (constrained) energy storage.

The input-output behavior of the RST can be described by the quadratic function (1b) for $m \in \{\text{rst}\}$. The input power of the RST is bounded by (1d) for $m \in \{\text{rst}\}$. The dynamics of the RST are assumed to satisfy a thermal energy balance given by
\[ C_{\text{rst}} \frac{dT_{\text{rst}}}{dt} = (P_{\text{rst}} - \beta_1 (T_{\text{rst}} - \beta_2 T_{\text{amb}})), \]
where $C_{\text{rst}}$ is the heat capacity of the RST and its contents, $P_{\text{rst}}$ is the thermal power where negative values indicate cooling, $\beta_1$ is the heat transfer coefficient between the RST and the environment, $\beta_2$ is an insulation coefficient, $T_{\text{rst}}$ is the temperature inside the RST and $T_{\text{amb}}$ is the ambient temperature (which is assumed to be constant). We can represent the RST model in terms of stored energy by defining the thermal energy relative to the ambient temperature $E_{\text{rst}} = C_{\text{rst}}(T_{\text{rst}} - T_{\text{amb}})$. By doing so and by making a forward Euler approximation of (17) with step size $\tau > 0$, the dynamics can be represented by (1c) for $m \in \{\text{rst}\}$ with $A_{\text{rst}} = 1 - \frac{\tau}{C_{\text{rst}}}$ and $B_{\text{rst}} = -\tau$ for all $k \in K$.

3.2 HVB Equivalent Circuit Model

The HVB dynamics are modeled with a battery equivalent circuit model shown in Fig. 2. We consider the so-called Rint model as well as the so-called Thevenin model (He...
et al., 2012). The Rint model consists of a circuit with a constant voltage source and a resistor. This model is generally used for energy management due to its simplicity. However, it is also the least accurate HVB model (He et al., 2012). The Thevenin model is a Rint model with additionally a parallel RC network connected to it, thereby including the dynamics of charge acceptance. With the Thevenin model, the HVB undergoes a low resistance when the capacitor C1 is not charged and the resistance increases as the charge increases. The dynamics of the HVB with the Rint model are given by

$$\dot{q} = -I,$$

with q the HVB charge and I the HVB current and for the Thevenin model given by (18a) and

$$U_1 = \frac{1}{C_1}(I - \frac{q}{C_1}),$$

where $U_1$ is the voltage over capacitor $C_1$. The HVB terminal voltage is given by

$$U_b = U_{oc} - R_0 I - U_1$$

for the Thevenin model and for the Rint model given by (18c) with $U_1 = 0$.

We can represent the HVB model in terms of stored energy by defining the energy in the HVB by $E_{hvb} = q U_{oc}$, the energy related to the capacitor by $E_1 = C_1 U_{oc} U_1$ and the battery power $P_{hvb,1} = U_{oc} I$. By doing so and by making a forward Euler approximation of (18) with step size $\tau > 0$, the dynamics can be represented by (1c) for $m \in \{hvb\}$ with $A_{hvb} = 1$, $B_{hvb} = -\tau$ for all $k \in K$ for the Rint model and with $A_{hvb} = \left[\begin{array}{cc} 1 & 0 \\ 0 & -\tau \end{array}\right]$ and $B_{hvb} = [-\tau |\tau]^T$ for all $k \in K$ for the Thevenin model. The input-output behavior of the HVB can be described by the quadratic function (1b) for $m \in \{hvb\}$ with $f_{m,k} = \frac{R_0}{C_1}$ for both the Rint as well as the Thevenin model and $a_{hvb,k} = 0$ for the Rint model and $a_{hvb,k} = \left[0 \ 1 \ -\frac{1}{\tau^2}\right]$ for the Thevenin model.

### 3.3 HVB model with aging

In (Wang et al., 2011), the following semi-empirical battery aging model is given for graphite-LiFePO4 battery cells

$$Q = B \cdot \exp\left(\frac{-P_{hvb}}{P_{max}}\right)$$

where $Q$ is the battery capacity loss in percentages, $R$ is the gas constant, $T$ is the battery temperature in degrees Celsius, which is assumed to be constant through active cooling, $A_0$ is the total Ah-throughput, $E_0$ is the activation energy given by

$$A_0 = \frac{1}{3600 U_{oc}} E_0 + \int_0^t |P_{hvb}| dt$$

where $E_0$ is the initial total energy throughput of the HVB system and $t$ is the time horizon over which the energy throughput is evaluated. In (Pham, 2015), a quasi-static approach is utilized where the rate of change of the $C_{rate}$ is neglected in order to derive the incremental capacity loss

$$\frac{dQ}{dt} = h(P_{hvb}) \frac{dQ}{dt}$$

with

$$h(P_{hvb}) = \left[\begin{array}{c} 1 \\ -\tau \end{array}\right]$$

It should be noted that (19) is the solution to (20), as was shown in (Pham, 2015). By making a forward Euler approximation of (20a) with step size $\tau > 0$, the battery capacity loss in discrete time is given by

$$Q_{k+1} = Q_k + \tau h(P_{hvb,k}) Q_{k+1}.$$  

Note that $h(\cdot)$ is not necessarily a convex function depending on the parameterizing of $B$ as function of the $C$-rate. Therefore, the battery capacity loss (21) can be approximated with (11) by taking the convex function

$$\alpha(P_{hvb,k}) = p_1 \exp(p_2 P_{hvb,k}) + p_3 \exp(p_4 P_{hvb,k})$$

such that $\alpha(P_{hvb,k}) P_{hvb,k}^2$ is $h(P_{hvb,k})$. The parameters $p_1, p_2, p_3, p_4$ are obtained with a least-squares fitting technique. The results are shown in Fig. 3 where $h(P_{hvb,k})$ and $\alpha(P_{hvb,k}) P_{hvb,k}^2$ are shown together with the absolute error between the two functions.

### 4. SIMULATION RESULTS

In this section, we analyze the effect of using the HVB model with the dynamics of charge acceptance and the

![Fig. 2. Equivalent circuit models](image)

![Fig. 3. Convex approximation of the aging factor.](image)
Fig. 4. The Pan-European driving cycle.

HVB aging constraint on the CVEM solution by means of a simulation study. To do so, the optimization problem given in (1) is solved for part of a Pan-European driving cycle with length $K = 20000$ s. The power request at the wheels and the engine speed for this driving cycle are given in Fig. 4.

4.1 Case studies

To study the effect of different battery models on CVEM, we define the following cases with increasing complexity:

1. In this case, the battery is not used, i.e., $P_{hvb} = 0$. Moreover, the temperature in the refrigerated semi-trailer is fixed at a constant temperature (namely, its upper bound) requiring a constant thermal power of $P_{th} = \beta_1(T_{th} - T_{amb})$.

2. In this case, one decision variable is added with respect to Case 1, namely the battery storage power $P_{hvb}$. The temperature in the refrigerated semi-trailer is still fixed at the upper bound. This case has one state, being the battery state-of-energy $E_{hvb}$.

3. In this scenario, another decision variable is added with respect to Case 2, namely the thermal power $P_{th}$. The thermal energy in the refrigerated semi-trailer is no longer fixed but constrained in an interval. This case has two energy states $E_{hvb}$ and $E_{th}$.

Note that only for Case 2 and Case 3 the HVB is used so that we can associate the optimization problem with the HVB Rint model or the HVB Thevenin model introduced in Section 3.2. Moreover, with the solution method proposed in Section 2.4, the optimization problem for Case 2 and Case 3 can be solved with the HVB Rint model together with the constraint on battery aging (11). To avoid disclosing confidential information, (19) is scaled with a constant factor but within a realistic range. Furthermore, the battery parameters $U_{oc}$, $R_0$, $R_1$ and $C_1$ are optimized to give the best approximation of a high-fidelity simulation model where, in general, the resistance $R_0$ is higher for the Rint model compared to the Thevenin model due to its reduced accuracy.

4.2 Results

In Fig. 5, the HVB power and RST power for Case 3 and for each of the three battery models, i.e., the Rint model, the Thevenin model and the Rint model with a constraint on aging are shown for a small part of the driving cycle. This figure clearly shows the charge acceptance dynamics in the Thevenin model. At short peaks, the Thevenin model can have a larger HVB power than the Rint model, i.e., the internal resistance $R_0$ is lower and the influence of $R_1$ is small when the capacitor is not charged. However, at longer positive/negative power intervals, the capacitor $C_1$ has a higher influence such that the Thevenin model effectively has a higher resistance than the Rint model, resulting in lower HVB powers. As less energy (power) is stored in the battery, more energy (power) is stored in the RST. The effect of charge acceptance on the HVB power and the RST power is still very limited though.

The effect of the constraint on battery aging is more pronounced. This is due to the fact that the battery lifetime without a constraint on aging is significantly shorter than with a constraint on aging, i.e., for this driving cycle, 0.11% of battery capacity loss is allowed while without aging constraint $^1$, the battery capacity loss was 0.29%. This is also shown in Fig. 6 where $Q_{th} = 1\%$. Here, the black line visualizes the imposed constraint on battery aging. This results in the battery power being limited in order to preserve battery life as is shown in Fig. 5. It can also be noted that large battery powers are reduced much more in magnitude than smaller battery powers, which is explained by Fig. 3, where it can be seen that larger battery powers have an exponentially larger

$^1$ The constraint of 0.11% is based on the desire to complete the drive cycle of Fig. 4 1000 times before reaching a capacity loss of 20%, while not changing the solution of the optimal control problem. It should be noted that for the same energy management strategy, the capacity decreases more initially than during the 1000th repetition of the drive cycle.
5. CONCLUSION

In this paper, we added battery charge acceptance limitations and battery aging limitations to the complete vehicle energy management problem. The problem has been solved using a distributed optimization approach for a case study of a hybrid heavy-duty vehicle, equipped with a refrigerated semi-trailer for two different battery models. The first battery model takes charge acceptance into account by adding an additional energy state to the optimization problem. The second battery model includes aging constraints. This larger difference in temperature and SoC between the two battery models shows the true benefit of complete vehicle energy management.

Table 2. Fuel consumption reduction.

<table>
<thead>
<tr>
<th>Case 2 vs. Case 1</th>
<th>Rint model</th>
<th>Thevenin model</th>
<th>Rint model and aging constraint</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.091</td>
<td>0.089</td>
<td>0.089</td>
</tr>
</tbody>
</table>

Fig. 7. RST temperature and SoC for the three different battery models.

The effect on battery aging. This larger difference in battery power results in a larger difference in RST power as well, as seen in Fig. 5.

Finally, in Fig. 7, the state-of-charge (SoC) and RST temperature are shown for the complete driving cycle. This figure clearly shows that with battery aging, substantially more energy is put into the RST and less energy is drawn from the battery compared to the battery models without constraint on aging. This also has a significant effect on the fuel reduction given in Table 2. For Case 1 and Case 3 with respect to Case 2. For Case 2, the fuel reduction with the Rint model is not significantly different than with the Thevenin model. The fuel reduction by taking into account the constraint on battery aging is, however, significantly lower, i.e., 8.61% instead of 9.4% This is caused by the fact that aging limitations cause less energy to be stored in the high-voltage battery. For Case 3 however, energy can also be stored in the refrigerated semi-trailer, smart control of this refrigerated semi-trailer leads to an additional fuel reduction 0.53% in the case where battery aging is incorporated, while it was only 0.09% when battery aging was not considered. In other words, the drop in the fuel consumption reduction caused by battery aging constraints can be partly compensated by smart control of other energy buffers, which shows the true benefit of complete vehicle energy management.

REFERENCES


Global Solutions to the Complete Vehicle Energy Management Problem via Forward-Backward Operator Splitting

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Abstract—Complete Vehicle energy Management (CVEM) aims to minimize the energy consumption of all subsystems in a vehicle. We consider the case where the subsystems consist of energy buffers with linear dynamics and/or energy converters with quadratic power losses. In this paper, we show the existence of only global solutions for the CVEM optimal control problem and propose a reformulation of this problem so that it can be solved using a Forward-Backward splitting algorithm for nonconvex optimization problems. The regularization properties inherent to this algorithm allow us to solve CVEM cases that were difficult using other approaches as dual decomposition.

1. INTRODUCTION

Improving energy efficiency of vehicles is an important topic of research for the automotive industry. While for traditional and hybrid vehicles, a better energy efficiency leads to lower emissions, the main motivation to improve efficiency of electric vehicles is that it will extend the range of the vehicle and thereby mitigating range anxiety. Range anxiety is the concern experienced by the user to be unable to reach a final destination with the current energy [1]. An energy management strategy (EMS) aims at reducing the energy consumption by an optimal distribution of power among the powetrain components (to provide the required power to the wheels) and the auxiliary systems. In essence, the EMS is the solution to an optimal control problem and optimization techniques for energy control on hybrid vehicles have been discussed in [2], [3]. A recent trend in this research area is to consider all the energy consumers inside the vehicle and this concept is known as Complete Vehicle Energy Management (CVEM) [4]. CVEM requires the optimal control problem to be scalable, as it requires a larger number of subsystems to be connected to the power network compared to earlier solutions for EMS.

Static optimization techniques have emerged as tractable approaches to tackle the CVEM problem, see, e.g., [5]–[8]. In particular, [6] has incorporated the control of auxiliary systems into the EMS and [5] has used convex relaxations to guarantee optimality of the solution. However, due to the use of centralized optimization methods, those approaches are not flexible in the sense that subsystems cannot be easily added or removed. The research presented in [7] uses a game-theoretic approach to solve the CVEM problems in a decentralized manner, where all the subsystems share a limited amount of information and are able to take some decisions autonomously. Global optimality of the centralized optimal control problem is not guaranteed.

A static distributed optimization approach for CVEM is presented in [8]. The method proposed in these papers are scalable and flexible. The main idea is to use dual decomposition to split the original optimal control problem into several simple problems related to interconnected subsystems. As a result, the computational time is drastically reduced and adding and removing subsystems becomes easy. However, several open questions exist related to numerical aspects of the algorithms proposed to solve the distributed optimization problem. For instance, [8] proposes a second-order dual update for which the convergence of the algorithm has not been formally proven. Additionally, the approach requires components to be described with linear dynamics and quadratic energy conversion models. In case the energy conversion is almost linear, the distributed optimization approach of [8] becomes ill conditioned, causing the algorithm to have difficulties to converge.

In this paper, we design an efficient algorithm with convergence guarantee for the CVEM problem, based on the Forward-Backward (FB) operator splitting method [9, §5.1]. First, we show that the non-convex CVEM problem only has global optimal solutions. Then, by taking advantage of the globality of the solutions, we massage the FB splitting method onto the CVEM problem to obtain a parallelizable static optimization algorithm. The convergence analysis of the proposed algorithm is based on a recent result on projected-gradient dynamics for non-convex optimization problems [9, §5.3]. Remarkably, the use of this projection method introduces regularization that prevents ill-conditioning of the optimization problem. This means that convergence is possible even if linear models are used to describe the power consumption of the vehicle subsystems.

Nomenclature: \( \mathbb{R} \) denotes the set of real numbers, \( \mathbb{R}_+ \) the set of non-negative real numbers, and \( \mathbb{R} := \mathbb{R} \cup \{+\infty\} \) the set of extended real numbers. For \( a, b \in \mathbb{R} \), a significant strict inequality that is denoted by \( a < b \) implies that \( a \) is much less than \( b \). The matrices \( 0 \) and \( I \) denote matrices with all elements equal to 0 and 1, respectively, and the matrix \( I \) denotes the identity matrix. To improve clarity, we sometimes add the dimension of these matrices as subscript. Furthermore, \( \text{diag}(A_1, \cdots, A_N) \) denotes a block-diagonal matrix with \( A_1, \cdots, A_N \) as diagonal blocks. Given \( N \) vectors \( x_1, \ldots, x_N \in \mathbb{R}^n \), we denote \( \text{col}(\{x_i\}_{i=1}^N) = [x_1^\top \ldots x_N^\top]^\top \). The short hand notation \( \{x_{p,q}\} \) is used to
denote $\{x_{pq}\}_{p \in P, q \in Q}$.

Given a set $S \subseteq \mathbb{R}^n$, the mapping $\mathbf{i}_S : \mathbb{R}^n \to \{0, +\infty\}$ denotes the indicator function satisfying $\mathbf{i}_S(x) = 0$ if $x \in S$ and $\mathbf{i}_S(x) = +\infty$ if $x \notin S$, and set-valued mapping $\mathbf{N}_S : \mathbb{R}^n \leftrightarrow \mathbb{R}^n$ denotes the normal cone operator satisfying $\mathbf{N}_S(x) = \{v \in \mathbb{R}^n \mid \sup_{x \in S} v^\top(x - x) \leq 0\}$ if $x \in S$ and $\mathbf{N}_S(x) = \emptyset$ if $x \notin S$. The mapping $\mathbf{proj}_S : \mathbb{R}^n \to S$ for a closed set $S \subseteq \mathbb{R}^n$ denotes the projection onto $S$, i.e., $\mathbf{proj}_S(x) = \text{argmin}_{y \in S} \|y - x\|^2$.

For a function $\psi : \mathbb{R}^n \to \mathbb{R}$ with $\text{dom}(\psi) := \{x \in \mathbb{R}^n \mid \psi(x) < +\infty\}$, the subdifferential set-valued mapping $\partial \psi : \text{dom}(\psi) \rightrightarrows \mathbb{R}^n$ is defined as $\partial \psi(x) := \{v \in \mathbb{R}^n \mid \psi(z) \geq \psi(x) + v^\top(z - x) \text{ for all } z \in \text{dom}(\psi)\}$. In case $\psi$ is continuously differentiable, the subgradient is equal to its gradient, i.e., $\partial \psi(x) = \nabla \psi(x)$.

II. CVEM AS OPTIMAL CONTROL PROBLEM

In this section, we first discuss a mathematical description of a power network model for CVEM. Later, we will formulate the discrete optimal control problem for CVEM, which will turn out to be non-convex, and we will show that all the solutions to this non-convex optimal control problem are global optimal solutions. Finally, we will propose a convenient equivalent formulation of the problem that will be exploited in Section III to find an optimization algorithm based on operator splitting techniques.

A. Power Network Model for CVEM

The CVEM problem aims to minimize the energy consumption for a network of subsystems $m \in M := \{1, \ldots, M\}$, where $M$ is the total number of subsystems. These subsystems are composed of an energy converter possibly combined with an energy buffer. For instance, considering a battery, the capacity of storing energy is modeled as an energy buffer, while the power losses produced during the transformation of chemical energy into electrical energy in the battery are represented by an energy converter.

A power network for CVEM is schematically depicted in Fig. 1. It can be noted that the energy buffers are always connected to energy converters by the power input $u_{m,k}$. On the other hand, the converters are connected to each other according to a specific topology via the network nodes $j \in J := \{1, \ldots, J\}$, where $J$ is the total number of nodes in the network. In this case, power outputs $y_{m,k}$ and inputs $u_{m,k}$ of the converters could be directly connected to a network node. Every node $j \in J$ can have a known exogenous load signal $w_{j,k}$ given for each time instant $k$. The following assumption on the network topology is considered in this paper.

Assumption 1: The power network has a tree structure topology, i.e., every subsystem is connected to only one node, and two consecutive nodes are always bridged by an individual energy converter. Moreover, only converters can be connected directly to a network node. □

Note that, albeit of its simplicity, the network presented in Fig. 1 contains the essential features described in Assumption 1, which can be found in more complicated power networks.

Fig. 1: Power network for CVEM.

B. Optimal Control Problem

The CVEM problem is an optimal control problem that aims to minimize the total aggregated energy consumption of the components over a time horizon $\mathcal{K} := \{0, 1, \ldots, K - 1\}$, while considering the interaction of all the components interconnected in the power network. Thus, the optimal control problem is given by

$$\min_{\{x_{m,k}, u_{m,k}, y_{m,k}\}} \sum_{m,k \in \mathcal{M}} \sum_{j \in \mathcal{K}} a_{m,k} y_{m,k} + b_{m,k} u_{m,k}, \quad (1a)$$

where $x_{m,k} \in \mathbb{R}^{n_{m,k}}$ are the states, $u_{m,k} \in \mathbb{R}$ are the (scalar) inputs and $y_{m,k} \in \mathbb{R}$ are the (scalar) outputs of the converter in subsystem $m \in \mathcal{M}$, while $a_m \in \mathbb{R}_+$ and $b_m \in \mathbb{R}$ are coefficients to define the objective function. A typical objective in CVEM is to minimize the fuel consumption, in which case assume the subsystem $m = 1$ corresponds to the combustion engine and $y_{1,k}$ denotes the chemical fuel power flow at time $k \in \mathcal{K}$. In this case, only $a_1 = 1$, while all other $a_m$ and $b_m$ are zero.

The objective function (1a) is to be solved subject to a quadratic equality constraint that describes the input-output behavior of each converter, i.e.,

$$y_{m,k} = q_{2,m} u_{m,k} + q_{1,m} y_{m,k} + q_{0,m}, \quad (1b)$$

where $q_{0,m} \in \mathbb{R}$, $q_{1,m} \in \mathbb{R}$ and $q_{2,m} \in \mathbb{R}_+$ are efficiency coefficients of the converter $m \in \mathcal{M}$, and subject to the linear system dynamics of the energy buffer

$$x_{m,k+1} = A_m x_{m,k} + B_m u_{m,k}, \quad (1c)$$

for all $k \in \mathcal{K}$ with $A_m \in \mathbb{R}^{n_{m,k} \times n_{m,k}}$ and $B_m \in \mathbb{R}^{n_{m,k} \times 1}$. The initial state $x_{m,0}$ of the storage device is assumed to be given and the inputs and states are subject to

$$\underline{x} \leq x_{m,k} \leq \overline{x}, \quad \underline{u} \leq u_{m,k} \leq \overline{u}, \quad (1d)$$

where for all $m \in \mathcal{M}$ the given state and input bounds are respectively $\underline{x}_m, \overline{x}_m \in \mathbb{R}^{n_{m,k}}$ and $\underline{u}_m, \overline{u}_m \in \mathbb{R}$. In [8], it has been shown that quadratic static models for energy converters and linear dynamical models for buffers are an adequate approximation to describe typical components in CVEM.

The interaction between the subsystems in the power network is given by the power balance at each node $j \in \mathcal{J}$. We distinguish between energy conserving nodes, given by

$$\sum_{m \in \mathcal{M}} c_{j,m} y_{m,k} + d_{j,m} u_{m,k} + w_{j,k} = 0, \quad (1e)$$

for $j \in \mathcal{J} \setminus \{1, 2, \ldots, J\}$ and energy dissipating nodes

$$\sum_{m \in \mathcal{M}} c_{j,m} y_{m,k} + d_{j,m} u_{m,k} + w_{j,k} \leq 0, \quad (1f)$$

for $j \in \mathcal{J} \setminus \{1, 2, \ldots, J\}$.
for $j \in J_d = \{ J_c + 1, J_c + 2, \ldots, J \}$. Observe that $J_c \cap J_d = \emptyset$ and $J_c \cup J_d = J$. In these expressions, $c_{j,m}$ is 1 if the correspondent power signal $y_{m,k}$ is connected to the node $j$ and 0 otherwise. The constant $d_{j,m}$ is $-1$ if the respective power signal $u_{m,k}$ flows into node $j$, it is 1 if the power flows out of node $j$, and 0 if the respective signal is not connected to the node. It should be noted that dissipating nodes $(1f)$ exist, e.g., because mechanical braking can be modeled as a power dissipation in a node. Finally, due to Assumption 1, we have that for every $m \in \mathcal{M}$, $c_{j,m} = 1$ for only a single $j \in \mathcal{J}$.

C. Global Solutions to the Nonconvex CVEM Problem

The optimal control problem (1) is non-convex due to $(1b)$. This might cause solvers to get stuck in a local minimum. In this section, we show that all (local) solutions to (1) are global solutions under very mild conditions.

To show that (1) has only global solutions, we relax the inequality constraint $(1b)$ to an inequality constraint, i.e.,

$$\frac{1}{2}q_{2,m}u_{m,k}^2 + q_{1,m}u_{m,k} + q_{0,m} = y_{m,k} \leq 0 \quad (1b')$$

for $m \in \mathcal{M}$ and $k \in \mathcal{K}$, allowing us to define a relaxed optimal control problem

$$\min_{\{u_{m,k}, x_{m,k}, z_{j,m}\}, m \in \mathcal{M}} \sum_{m \in \mathcal{M}} \sum_{k \in \mathcal{K}} a_{m}y_{m,k} + b_{m}u_{m,k} \quad \text{s.t.} \quad (1b'), (1c) - (1f). \quad (2)$$

The discrete optimal control problem (2) is convex, thus has only global solutions. The optimal value of the original problem (1) and of the convex relaxation (2) satisfy

$$\rho^R \leq \rho^NC, \quad (3)$$

where $\rho^R$ and $\rho^NC$ denote the optimal value of the convex relaxation (2) and the non-convex optimal control problem (1), respectively, because the convex relaxation has a larger feasible set due to $(1b')$.

The next Theorem states that for this particular optimal control problem and its relaxation, it holds that, $\rho^R \leq \rho^NC$ and that (2) always has a solution that satisfies $(1b')$ with equality, thus it satisfies $(1b)$ and solves (1).

**Theorem 1:** Assume that the bounds $\{u_{m,k}\}_{m \in \mathcal{M}}$ and $\{\pi_{m}\}_{m \in \mathcal{M}}$ are finite and that there exists a feasible point $\{u_{m,k}, y_{m,k}, x_{m,k}, z_{j,m}\}_{m \in \mathcal{M}, k \in \mathcal{K}}$ for (2) with strict inequalities $(1b')$ and (1d). Then, an optimal solution to (2) satisfies (1b') with equality, $\rho^R = \rho^NC$, and (1) only has global optimal solutions.

D. An Equivalent Formulation of the CVEM Problem

In this subsection, we will reformulate the optimal control problem (1) into an equivalent form that will later be recast as a static optimization problem that is suitable to be solved with the operator splitting technique presented in Section III.

The reformulation of (1) considers three main steps that are described and justified as follows:

1) Substitution of the quadratic equality constraint $(1b)$ into (1a), (1e), (1f). This substitution aims to reduce the number of decision variables by the elimination of $\{y_{m,k}\}_{m \in \mathcal{M}, k \in \mathcal{K}}$ from the optimal control problem.

2) Conversion of the inequality constraint $(1f)$ into an equality constraint using slack variables $\{z_{j,k}\}_{j \in \mathcal{J}, k \in \mathcal{K}}$ of which some are constrained to $z_{j,k} = 0$ for all $j \in \mathcal{J}$, and $k \in \mathcal{K}$. This yields that the Lagrangian of the optimal control problem (4) is a mapping from $\mathbb{R}$ to $\mathbb{R}$, which is necessary for the algorithm that will be proposed Section III, see [9, §5].

3) Introduction of quadratic penalty terms associated to the equality constraints $(1e)$ and $(1f)$. This is done without removing these constraints. The resulting Lagrangian function of the optimal control problem can be seen as an augmented Lagrangian function. The advantage of this formulation is that it introduces regularization to the optimization procedure, thereby improving the convergence properties of the algorithm [10, §3.2.1, §4.2].

The equivalent optimal control problem obtained as a result of the above steps is given by

$$\min_{\{u_{m,k}, x_{m,k}, z_{j,m}\}, m \in \mathcal{M}} \left\{ \begin{array}{l}
\sum_{m \in \mathcal{M}} \sum_{k \in \mathcal{K}} p_m(u_{m,k}) + \sum_{j \in \mathcal{J}} \sum_{m \in \mathcal{M}} r_{j,m}(u_{m,k}) + \frac{\sigma_j}{2} z_{j,k}^2 \\
\text{s.t.} \quad (1c), (1d), \quad \sum_{m \in \mathcal{M}} r_{j,m}(u_{m,k}) + \frac{\sigma_j}{2} z_{j,k}^2 = 0 \quad \forall j \in \mathcal{J},
\end{array} \right. \quad (4a)$$

$$z_{j,k} = 0 \quad \forall j \in \mathcal{J}, k, \quad (4b)$$

where $\sigma_j \in \mathbb{R}_+$ are coefficients that weight the penalty terms in (4a). Furthermore,

$$p_m(u_{m,k}) = a_m \left( \frac{1}{2}q_{2,m}u_{m,k}^2 + q_{1,m}u_{m,k} + q_{0,m} \right) + b_{m}u_{m,k}, \quad (4c)$$

$$r_{j,m}(u_{m,k}) = c_{j,m} \left( \frac{1}{2}q_{2,m}u_{m,k}^2 + q_{1,m}u_{m,k} + q_{0,m} \right) + d_{j,m}u_{m,k} + w_j. \quad (4d)$$

It should be noted that constraint (4c) acts only on the non dissipating nodes $\mathcal{J}_c$, which means that the equality constraint $(1e)$ is embedded in (4b) for this formulation.

The equivalence of the discrete time optimal control problems (1) and (4) is the main result of this section. In order to present this result, we will indicate a constraint qualification (CQ) that (4) satisfies. For a complete survey of CQ for non-linear programming see [11]. In the following lemma, we present a condition to guarantee that (4) satisfies a linear independence CQ (LICQ).

**Lemma 1:** The feasible set of the discrete optimal control problem (4) satisfies LICQ, if for all $m \in \mathcal{M}$ the bounds on $u_{m,k}$ in (1d) satisfy

$$y_{m,k} > \frac{-\pi_{m}}{w_{m,k}} \quad \text{or} \quad \pi_{m} < \frac{-y_{m,k}}{w_{m,k}}. \quad (7)$$

The condition to guarantee LICQ of discrete optimal control problem (4) presented in this lemma can be tested a priori via a simple inspection of the power bounds for all the subsystems. For realistic applications, it typically holds that...
|q_{2,m}| < |q_{1,m}|$, which suggests that (7) is a mild condition. The satisfaction of LICQ by the optimal control problem (4) indicates that its critical points are regular. This will be exploited in the following Theorem to show the equivalence between the problem formulations (1) and (4) in terms of the Karush-Kuhn-Tucker (KKT) conditions for optimality.

**Theorem 2**: The optimization problems (1) and (4) have the same necessary conditions for optimality, and same global minimizers, if the conditions in Lemma 1 are satisfied.

### III. A Forward-Backward Algorithm for CVEM

In the previous section, we showed that the CVEM problem (1) only has global solutions, thus solvers cannot get stuck in local minima. Moreover, it was demonstrated that (4) is an equivalent formulation to (1). In this section, we will take advantage of the previous results to propose a method to solve the CVEM problem (1) using an operator splitting approach. To achieve this we will recast the equivalent optimal control problem (4) into a non-convex static optimization problem. Then, we will split the cost function of this static optimization problem as the sum of a non-convex function and a convex set-valued function. This allows the forward-backward splitting method for non-convex optimization problems to be applied.

#### A. Reformulation as a Static Optimization Problem

To reformulate the optimal control problem (4) as a static optimization problem, we define the following vectors

\[
u_m = \text{col}(\{u_{m,k}\}_{k \in \mathcal{K}}) \in \mathbb{R}^K, \quad (8a)
\]

\[
x_{m} = \text{col}(\{x_{m,k}\}_{k \in \mathcal{K}}) \in \mathbb{R}^{K_{n_{m}}}, \quad (8b)
\]

\[
z_{j} = \text{col}(\{z_{j,k}\}_{k \in \mathcal{K}}) \in \mathbb{R}^K, \quad (8c)
\]

\[
w_{j} = \text{col}(\{w_{j,k}\}_{k \in \mathcal{K}}) \in \mathbb{R}^K, \quad (8d)
\]

for $m \in \mathcal{M}$ and $j \in \mathcal{J}$. This notation allows us to write the solutions to (1c) in the compact form

\[
x_{m} = \Phi_{m}x_{0,m} + \Gamma_{m}u_{m}, \quad (9)
\]

with matrices $\Phi_{m} \in \mathbb{R}^{K_{n_{m}} \times n_{m}}$ and $\Gamma_{m} \in \mathbb{R}^{K_{n_{m}} \times K^2}$, and write (1d) as

\[
x_{m} \leq x_{m} \leq \bar{x}_{m} \text{ and } y_{m} \leq y_{m} \leq \bar{y}_{m} \cdot \quad (10)
\]

Now, by exploiting (8) - (10), we rewrite the discrete optimal control problem (4) as the following static optimization problem:

\[
\begin{align*}
\min_{u_{m,j}} & \sum_{m \in \mathcal{M}} P_{m}(u_{m}) + \sum_{j \in \mathcal{J}} \frac{1}{2} \left\| \sum_{m \in \mathcal{M}} R_{m,j}(u_{m}) + \frac{1}{2} z_{j}^{2} \right\|^{2}, \\
s.t. & \sum_{m \in \mathcal{M}} R_{m,j}(u_{m}) + \frac{1}{2} z_{j}^{2} = 0, \quad \text{for all } j \in \mathcal{J}, \quad (11a) \\
& z_{j} = 0, \quad \text{for all } j \in \mathcal{J}, \quad (11b) \\
& u_{m} \in [\underline{u}_{m}, \bar{u}_{m}], \quad \text{for all } m \in \mathcal{M}, \quad (11c)
\end{align*}
\]

where

\[
P_{m}(u_{m}) = \left\{ \begin{array}{ll}
\Phi_{m}x_{0,m} + \bar{u}_{m} & \text{if } u_{m} \geq \underline{u}_{m}, \\
\bar{u}_{m} & \text{otherwise},
\end{array} \right. \quad (12a)
\]

\[
R_{m,j}(u_{m}) = c_{m,j}y_{m}(u_{m}) + d_{m,j}u_{m} + w_{j}, \quad (12b)
\]

\[
\Omega_{m} := \{u_{m} \in \mathbb{R}^K | \Phi_{m}x_{0,m} + \Gamma_{m}u_{m} \leq \bar{x}_{m}, \Gamma_{m}u_{m} \leq y_{m}, \Gamma_{m}u_{m} \leq \bar{y}_{m} \}, \quad (12c)
\]

in which

\[
\begin{align*}
Y_{m}(u_{m}) &= \text{col}(\{\frac{1}{2} q_{2,m}u_{m,j}^{2} + q_{1,m}u_{m,k} + q_{0,m}\}_{k \in \mathcal{K}}). \quad (13)
\end{align*}
\]

This convenient reformulation shows a non-convex static optimization problem that will be used to design a parallelizable algorithm to solve the CVEM problem (1), based on the FB operator splitting method [9, §5.1].

#### B. KKT Conditions

Operator splitting methods [12, §26] and [13] can be used to find zeros of (set-valued) mappings. In constrained optimization theory, these splitting methods are used to find the points that satisfy the KKT conditions. In particular, constraints (1d) will be embedded in the optimization problem using indicator functions, which will make the optimization problem non-smooth. In doing so, the KKT conditions of the static optimization problem (11) can be represented as a set-valued mapping, which is shown in this subsection, so that (candidate) minimizers can be found using operator splitting methods.

Let us first introduce vectors $u = \text{col}(\{u_{m}\}_{m \in \mathcal{M}})$, $z = \text{col}(\{z_{j}\}_{j \in \mathcal{J}})$, $\lambda = \text{col}(\{\lambda_{j}\}_{j \in \mathcal{J}})$, and the Lagrangian function of (11), which is given by

\[
L(u, z, \lambda) = \sum_{m \in \mathcal{M}} P_{m}(u_{m}) + \sum_{j \in \mathcal{J}} \left( \frac{1}{2} \left\| \sum_{m \in \mathcal{M}} R_{m,j}(u_{m}) + \frac{1}{2} z_{j}^{2} \right\|^{2} + \lambda_{j}^{T} \sum_{m \in \mathcal{M}} R_{m,j}(u_{m}) + \frac{1}{2} z_{j}^{2} \right), \quad (14)
\]

with Lagrange multipliers $\lambda_{j} \in \mathbb{R}^{K_{j}}$, $j \in \mathcal{J}$. The specific design of the Lagrangian function (14) is connected to the splitting algorithm that will be presented in Section III-C. Namely, the linear part of the feasible set is expressed as indicator functions while the non-linear and possibly non-convex part of the problem (11) is embedded in the continuously differentiable function $h$. The linear part of the feasible set is also separable per $m \in \mathcal{M}$, which will make the algorithm parallelizable.

The KKT conditions of the augmented optimization problem (11) can be expressed in terms of the subdifferential of the Lagrangian function (14) as $0 \in \partial L(u, z, \lambda)$, with

\[
\begin{align*}
\partial L(u, z, \lambda) &= \begin{bmatrix}
\text{col}(\{F_{m}(u_{m}, z_{j}, \lambda_{j})\}_{m \in \mathcal{M}}) \\
\text{col}(\{G_{m}(u_{m}, z_{j}, \lambda_{j})\}_{m \in \mathcal{M}}) \\
\text{col}(\{-A_{j}(u_{m}, z_{j})\}_{j \in \mathcal{J}}) \\
\text{col}(\{N_{m}(u_{m}, z_{j})\}_{m \in \mathcal{M}}) \\
0 \\
0 \\
\end{bmatrix} + \begin{bmatrix}
\text{col}(\{N_{m}(u_{m}, z_{j})\}_{m \in \mathcal{M}}) \\
\end{bmatrix}, \quad (16)
\end{align*}
\]
where \( \Omega = ( \prod_{m \in M} \Omega_m ) \times \{ 0 \}^{L_c} \) and
\[
F_m(u,z,\lambda) = \nabla P_m(u_m) + \sum_{j \in J} R_m(u_m)(\lambda_j + \sigma_j A_j(u,z_j)),
\]
\[
G_j(u,z,\lambda_j) = \text{diag}(z_j)(\lambda_j + \sigma_j A_j(u,z_j)),
\]
\[
A_j(u,z_j) = \sum_{m \in M} R_m(u_m) + \frac{1}{2} z_j^2,
\]
(17a) (17b) (17c)

in which
\[
\nabla P_m(u_m) = a_m I_K \nabla y_m(u_m) + b_m I_K,
\]
\[
\nabla R_m(u_m) = c_{j,m} \nabla y_m(u_m) + d_{j,m} I_K,
\]
\[
\nabla y_m(u_m) = \text{diag}(g_{2,m,v_m,k} + q_m I_K).
\]
(18a) (18b) (18c)

In (16), we characterized the KKT conditions related to the static optimization problem (11). Under certain regularity conditions, the points \( \{ u^*, z^*, \lambda^* \} \) satisfying \( 0 \in \partial L(u^*, z^*, \lambda^*) \) provide candidate minima of the optimization problem (11), see [10, §3.3.1] and [14] for a detailed discussion on this topic. In the theorem below, we formally state that \( 0 \in \partial L(u^*, z^*, \lambda^*) \) leads to global minimizers to the discrete time optimal control problem (1).

**Theorem 3:** Suppose conditions of Lemma 1 are satisfied, and sets \( \Omega_m \) for all \( m \in M \) are compact. Then, global solutions \( \{ u^*, z^*, \lambda^* \} \) to the discrete time optimal control problem (1) can be obtained from points \( \{ u^*, z^*, \lambda^* \} \) satisfying \( 0 \in \partial L(u^*, z^*, \lambda^*) \).

**C. Forward-Backward (FB) Splitting Algorithm**

In this section, we apply the FB splitting method presented in [9, §5.1], which will be used to find points \( \text{col}(u^*, z^*, \lambda^*) \) satisfying \( 0 \in \partial L(u^*, z^*, \lambda^*) \), thereby finding global minimizers of the optimal control problem (1). It should be noted that for convex optimization problems, FB splitting leads to the widely used proximal gradient method (see [12, §2.6.5]), while recent extensions of the FB splitting show application to solve non-convex problems [9], [15].

To see how FB splitting is applied here, note that the right-hand side of (16) consists of two terms: \( \nabla h \), which is the gradient of the continuously differentiable function (15), and \( N_{\Omega_R(x^*,z^*,\lambda^*)} \), which is the operator that contains the normal cones of the linear feasible set defined by constraints (11c) and (11d). Now let \( \Phi \) be a preconditioning matrix, given by
\[
\Phi := \text{diag}(\{ \alpha_m \}_{m \in M}, \{ \theta_j \}_{j \in J}, \{ \gamma_j \}_{j \in J})
\]
(19)

where \( \alpha_m, \gamma_j \) and \( \theta_j \) are positive real constants that define the step sizes of the proposed algorithm. The FB operator splitting method [9, §5.1], applied on the mapping in (16) with preconditioning \( \Phi \) reads as
\[
\omega^{i+1} = \text{proj}_{\Omega_R(x^*,z^*,\lambda^*)}(\omega^i - \Phi \nabla h(\omega^i)),
\]
(20)

with \( \omega = \text{col}(u,z,\lambda) \) and superscript \( i \in \{ 1, 2, \ldots \} \) denoting the \( i \)-th iteration. It should be noted that both the function \( \nabla h \) as well as the projection are highly structured. This allows for a parallelizable algorithm. Furthermore, the projections on \( \Omega_m \) can be implemented efficiently as a constrained least squares problem. The final algorithm is presented below.

**Algorithm 1: Forward-Backward Splitting**

**Initialization:**
Select step-sizes \( \alpha_m, m \in M, \gamma_j, j \in J \) and \( \theta_j, j \in J \); select regularization weights \( \sigma_j, j \in J \) and select initial values \( u_m^0, m \in M, z_j^0 = 0 \) for all \( j \in J \) and all \( i \in \mathbb{N}, z_j^i \in \mathbb{R}^K \) for all \( j \in J \) and \( \lambda_j^0 \in \mathbb{R}^K \) for all \( j \in J \).

**Iterate until convergence:**
\[
u_m^{i+1} = \text{proj}_{\Omega_m}(u_m^i - \alpha_m F_m(u^i, z^i, \lambda^i)),
\]
for all \( m \in M, z_j^i = z_j^i - \theta_j G_j(u^i, z^i, \lambda^i),
\]
for all \( j \in J \)
\[
\lambda_j^{i+1} = \lambda_j^i + \gamma_j \theta_j G_j(u^i, z^i, \lambda^i),
\]
for all \( j \in J \)

It remains to show how to choose the step sizes in Algorithm 1 so that the algorithm converges. In [9, Th. 5.3], it is shown that if \( \alpha_m, \gamma_j \) and \( \theta_j \) are chosen sufficiently small, the algorithm converges. For more details, the reader is referred to [9].

**IV. NUMERICAL EXAMPLE**

In this section, we will use the method presented in Section III-C to solve the CVEM optimal control problem for a parallel-hybrid electric vehicle, consisting of and internal combustion engine (ICE), electric machine (EM) and a high-voltage battery (HVB). For notational convenience, we use the set \( M = \{ \text{ice, em, hvb} \} \), instead of \( M = \{ 1, 2, 3 \} \), and the set \( J = \{ a, b \} \), instead of \( J = \{ 1, 2 \} \).

The power network for this study case is presented in Fig. 2. There exists two nodes \( J \) that describe the mechanical and electrical power balance in the vehicle. These nodes are bridged by the EG that acts as a single power converter to transform power between the electrical and mechanical domains. The power network also contains exogenous signals that represent the power consumed by auxiliary subsystems \( w_m = 1.5[kW] \) and the traction power request \( w_a = y_{req} \), which is presented in Fig. 3a. The presence of the mechanical braking power \( y_{br} \leq 0 \) in node \( b \) implies that it is a dissipative node, thus \( J_c = \{ a \} \) and \( J_d = \{ b \} \). The parameters that describe the subsystems are given in Table I and by selecting, \( d_{a,hvb} = d_{a,em} = -1, c_{a,hvb} = c_{a,em} = 1 \) and all other coefficients as zero. Thus, the power network is described by
\[
y_{hvb,k} + y_{em,k} + w_{a,k} = 0,
\]
(21)
\[\]
\[\]
\[
y_{ice,k} - u_{ice,k} + w_{hvb,k} = 0.
\]
(22)
For this case, the CVEM is formulated to minimize the energy consumed by the ICE, which implies that \(a \mu = 1\) while the other \(a m\) and all \(b m\) are zero. In order to obtain results that reflect the energy consumption only related to fuel, constraints on the initial and final charge states of the battery are imposed, i.e., \(x_{0, h b} = x_{h b, 0}\). It is important to remark that this constraint on the final state can be easily included in the feasible set \((12c)\) for \(m = h b\), without affecting the main results and the methodology presented in this paper. Finally, it is important to mention that the problem has been discretized using a sampling time \(\tau = 5(s)\), which leads to \(A_{h b, k} = 5\) and \(B_{h b, k} = 5\).

Given the fact that condition (7) of Lemma 1 can be easily verified for this example, we have that Theorem 3 states that Algorithm 1 gives a global solution. For this example, two possible solutions are presented in Fig. 3b and Fig. 3c, which are obtained using the same initial conditions \((w_{0, m} = 0, x_{0, h b} = 0\) and \(x_{h b, 0} = 0\) for all \(m \in M\) and \(j \in J\)) for Algorithm 1, but using different step sizes, see Table II. The energy consumed in both cases is \(9.889515[(J)/100(km)]\), which shows that both solutions are indeed global minimizers of the optimal control problem. The convergence of Solution 1 was obtained after 365 iterations of the algorithm, while Solution 2 converged after 36 iterations. This is a remarkable result due to the fact that the use of the dual decomposition approach proposed in [8], does not converge for this numerical example. The observed improvement in convergence is related to the regularization properties introduced by the projections performed in Algorithm 1.

V. CONCLUSIONS

In this paper, we have proven that the non-convex CVEM problem only has global optimal solutions under mild condi-

### REFERENCES

A Port-Hamiltonian Approach to Complete Vehicle Energy Management: A Battery Electric Vehicle Case Study

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In the current literature, the modelling and the OCP formulation are mainly obtained by applying power-based approaches [9]–[13]. The main idea of these approaches is to describe the behavior of the interconnected subsystems network in term of power interactions. In other words, each subsystem is normally represented by an energy buffer connected to a power converter. The buffer stores energy and it is modeled as a dynamic linear system, which often resembles an integrator. The power converter represents energy consumption and is modeled as a static non-linearity, normally approximated to a quadratic function from data measurements, which often lacks a physical interpretation. By considering only power and energy to model the subsystems, the number of decision variables in the OCP problem is reduced. Moreover, the models show low complexity which makes the CVEM problem tractable.

However, power-based approaches are difficult to use when the optimization variables cannot be easily described in terms of power. For instance, in [14], [15] attempts to unify CVEM and eco-driving (finding energy optimal velocity profiles) are presented. In this case, velocity becomes a decision variable and describing it in terms of power is a non trivial task. Other disadvantage of power-based approaches is that it is difficult to constrain physical variables that describe power in the model. For instance, imposing constraints on the battery power do not necessarily imply that the solution of the OCP is physically realizable. Power in the battery is described as the product of voltage and current, thus an optimal solution that satisfies power constraints not necessarily satisfies the bounds on voltage and currents. Additionally, power-based approaches do not always allow for an intuitive formulation of decomposable OCP, which are useful when distributed optimization technique are applied to approximate solutions. For instance, in [10] the concept of ‘sum of losses’ is introduced to formulate separable OCP for CVEM. Unfortunately, this concept is not always simple to apply.

In the paper, we propose an alternative modelling framework based on interconnected port-Hamiltonian (pH) systems [16], [17] aiming to overcome the previously discussed limitations of power-based approaches. In this modelling approach, each subsystem is represented as a dissipative dynamical system that inherently describes energy losses and changes of internal energy in the subsystem. The main contributions of this work are threefold. First, we propose to use pH representations to model networks for CVEM applications, thus unifying the power based concepts of energy buffers and power converters in a single dynamical
model, i.e., in terms of internal energy and power losses. Second, a systematic approach is proposed to formulate a decomposable OCP for CVEM, which is useful when distributed static optimization techniques are considered to solve the CVEM OCP. Third, an adaptation of the distributed optimization algorithm of [13] is presented to find solutions to the proposed pH CVEM OCP.

The remainder of this paper is organized as follows. In Section II, we briefly introduce the pH modeling theoretical background and use these concepts to model CVEM networks, which in Section III, allow us us to formulate decomposable OCP for CVEM with pH representations. In Section IV, a distributed static optimization problem to solve pH OCP for CVEM is summarized. The pH methodology for CVEM is applied to a case study presented in Section V. Finally, the conclusions are drawn in Section VI.

II. A PORT-HAMILTONIAN MODELLING APPROACH FOR CVEM

The main objective of this section is to present the pH modeling framework for CVEM and the formulation of decomposable OCP. Furthermore, we adapt the pH formalism to the holistic philosophy of CVEM presented in [8]. To achieve this, we briefly discuss the main concepts of pH representations. Later, the main features of networks of interconnected subsystems for CVEM are translated into a compatible description with pH models.

A. Port-Hamiltonian Representation

From a modeling perspective, a pH representation is a port-based modelling approach that considers a pair of conjugated variables in each port, which product represents power. This has been depicted in Fig. 1, where a generic graphical representation of a two-port pH system is presented. Although in literature there is a large amount of possible pH representations, we will only focus on linear subsystems, where a port-based modelling approach that considers a pair of conjugated variables in each port, which product represents power.

Consider $m \in \mathcal{M} := \{1, \ldots, M\}$ linear subsystem, where $\mathcal{M}$ is the total number of interconnected subsystems in the network. The internal energy of the subsystems is expressed as a quadratic function given by

$$\mathcal{H}_m(x_m) = \frac{1}{2} x_m^\top Q_m x_m,$$

where $x_m \in \mathbb{R}^n$ represents the state vector and $Q_m \in \mathbb{R}^{n \times n}$ is a symmetric positive semi-definite matrix, which is called energy matrix. This energy function is used in an input-state-output with feedthrough term pH representation [17], [16, §4]

![Fig. 1: Port-Hamiltonian (sub)system $\Sigma_m$](image)

of the $m$ subsystem, and is given by

$$\dot{x}_m = (J_m - R_m)Q_m x_m + B_m u_m$$

$$y_m = (B_m + 2P_m)^\top Q_m x_m + (M_m + S_m) u_m$$

where $x_m \in \mathbb{R}^n$ is the state vector and $u_m, y_m \in \mathbb{R}^2$ are the control input and output, respectively, given by

$$u_m = \begin{bmatrix} u_m^\text{in} \\ u_m^\text{out} \end{bmatrix} \quad \text{and} \quad y_m = \begin{bmatrix} y_m^\text{in} \\ y_m^\text{out} \end{bmatrix}.$$

From the port-Hamiltonian formalism in (2), we can describe the following power balance equation [16, §4]

$$\frac{\partial \mathcal{H}_m(x_m)}{\partial t} = x_m^\top L_m x_m + y_m^\text{in} u_m - y_m^\text{out} u_m$$

in which

$$L_m := \begin{bmatrix} Q_m R_m & Q_m P_m \\ P_m^\top Q_m & S_m \end{bmatrix}.$$

In case $L_m \succeq 0$, passivity of the system is guaranteed [18, §7.1]. Furthermore, an energy balance can be described by reorganizing and integrating both sides of (4) over the interval $[t_0, t_f]$, hence obtaining

$$\Delta E_m = E_m(t_f) - E_m(t_0) = \Delta \mathcal{H}_m + \mathcal{L}_m$$

with

$$\Delta E_m = \int_{t_0}^{t_f} y_m^\text{in} u_m \ dt = \int_{t_0}^{t_f} \begin{bmatrix} y_m^\text{in} & y_m^\text{out} \end{bmatrix} \begin{bmatrix} u_m^\text{in} \\ u_m^\text{out} \end{bmatrix} \ dt$$

$$\Delta \mathcal{H}_m = \mathcal{H}_m(x_m(t_f)) - \mathcal{H}_m(x_m(t_0))$$

$$\mathcal{L}_m = \int_{t_0}^{t_f} x_m^\top L_m x_m \ dt$$

where $\Delta E_m$ represents the net supplied energy, $\Delta \mathcal{H}_m$ is the change on internal energy and $\mathcal{L}_m$ describes the total energy losses. Moreover, let us define the set of source subsystems

$$S := \{m \in \mathcal{M} \mid E_m = 0\},$$

the set consumers

$$\mathcal{C} := \mathcal{M} \setminus S,$$

and the set of terminal subsystems

$$\mathcal{T} := \{m \in \mathcal{M} \mid E_m^\text{in} = 0\}.$$

A physical example of a source subsystem, which only has an output port, is chemical energy in a battery. On the other hand, mechanical brake is a physical example of a terminal system, which has only an input port.

B. Port-based Network Topology

The interconnections of pH systems have been studied in [19], where a methodology to describe networks of pH systems with the final goal to perform stability analysis has been proposed. Although in this paper we consider some ideas from the work presented in [19], we will deviate from this approach to propose a specialized description of network
topologies that capture specific features found in CVEM applications.

A tree structure port-based network topology is presented in Fig. 2, where each subsystem $m \in \mathcal{M}$ is depicted by a gray square. The interconnection between subsystems takes place through junctions $j \in \mathcal{J} := \{1, \ldots, J\}$, where $J \in \mathbb{N}$ is the total number junctions in the network. These junctions are power preserving interconnections that are described in terms of the conjugated input output variables as follows

$$\sum_{m \in \mathcal{M}} a_{j,m} y_m^{in} y_m^{out} + b_{j,m} y_m^{in} y_m^{out} = 0, \quad \text{for all } j \in \mathcal{J},$$

where $a_{j,m} = 1$ if the input port of subsystem $m$ is connected to junction $j$, and $a_{j,m} = 0$ otherwise. Similarly, $b_{j,m} = 1$ if the output port of subsystem $m$ is connected to junction $j$, and $b_{j,m} = 0$ otherwise.

In CVEM networks, there is a type of junction that can be decomposed into an additive node and an equality node. A physical interpretation of these type of nodes is observed in electrical interconnections between subsystems, e.g., Kirchhoff’s law of currents can be seen as an additive node, while Kirchhoff’s law of voltages is an equality node. Let us define the set of all the decomposable nodes in the network $\mathcal{J}_\text{d} := \{1, \ldots, J_\text{d}\}$, where $J_\text{d} \leq J$ is the total number of these type of junctions in the network. Thus, the power preserving equation (9) can be rewritten as

$$\sum_{m \in \mathcal{M}} [a_{j,m} 0] u_m + [0 b_{j,m}] y_m = 0, \quad \text{for all } j \in \mathcal{J}_\text{d},$$

where (10a) and (10b) are referred as additive node and an equality node, respectively.

Another type of power preserving interconnections observed in CVEM networks are power junctions. The power junction artifact introduced in this framework obviates the necessity to resemble the functionality of DC-DC converters in CVEM networks. It is well known that DC-DC converters can be represented as non-linear pH systems, e.g., see [20]. In fact, from steady-state approximations of the models proposed in [20] under the assumption that the converter is lossless, it is possible to prove that DC/DC converters satisfy

In order to keep consistency in the notation, it is possible to rewrite (9) as

$$\sum_{m \in \mathcal{M}} y_m^T \begin{bmatrix} a_{j,m} & 0 \\ 0 & b_{j,m} \end{bmatrix} u_m = 0, \quad \text{for all } j \in \mathcal{J}_\text{d};$$

where $\mathcal{J}_\text{d} := \{J_\text{d} + 1, \ldots, J\}$ is the set of all power junctions in the network.

III. OPTIMAL CONTROL PROBLEM FOR PH CVEM

In the first part of section, we first provide a general formulation for a continuous-time optimal control problem (OCP) for the pH CVEM modelling approach described in the previous section. Subsequently, we propose a physically insightful cost function that enables the decomposability of the proposed pH OCP for CVEM. Finally, we present a discrete-time OCP for pH CVEM.

The main objective of CVEM is to minimize the total energy consumed by a network of interconnected subsystems over a fixed time interval, subject to the dynamical behavior of each subsystem and bounds on the respective inputs, outputs, and states. From a mathematical perspective, this goal can described as an OCP. Hence, a general formulation of the pH OCP for CVEM is given by

$$\min_{\{x_m, u_m, y_m\} \in \mathcal{M}} \mathcal{J} \left( \{x_m, u_m, y_m\} \in \mathcal{M} \right)$$

subject to: (2), (10), (11),

$$x_m \leq x_m \leq x_m, \quad \text{for all } m \in \mathcal{M} \quad (12b)$$

$$y_m \leq u_m \leq y_m, \quad \text{for all } m \in \mathcal{M} \quad (12c)$$

$$y_m \leq y_m \leq y_m, \quad \text{for all } m \in \mathcal{M} \quad (12d)$$

Note that (12a) represents a general cost function that aims to describe the total energy consumed by the subsystems in the network. We will propose a physically insightful, and decomposable cost function for the OCP (12) below.

A. Power Consumption in pH CVEM

The physical interpretations provided by pH models in terms of internal energy and energy losses of the subsystems can be exploited to formulate a sensitive description of the energy consumed in the network. Thus, we could aim to maximize the change of internal energy in each source subsystem, which implies minimizing consumption, i.e., see (7b). This can be described by (12) if the cost function is defined as

$$\mathcal{J} = \sum_{m \in \mathcal{S}} -\Delta H_m.$$  

Note that the minus sign in (13) is necessary because (12) is defined as a minimization problem.

Interestingly, by decomposing the cost function (13) it is possible to obtain an equivalent expression that can be interpreted as minimizing the aggregated internal energy of all the consumers and the energy losses of all the subsystems. In order to see this, let us recall that power preserving interconnections are considered for all the junctions $j \in \mathcal{J}$.
in the network. Therefore, by integrating (9) for a given time interval we obtain the following energy balance

$$\sum_{m \in M} m_j m_j^{\text{em}} + b_j m_j^{\text{em}} = 0,$$ \hspace{1cm} (14)

for all \( j \in J \). Hence, it is possible to follow a recursive procedure where we substitute (6) and (14) into (13) to obtain the partially expanded function

$$\mathcal{F} = \sum_{m \in M} L_m - E_m^{\text{out}}$$

$$= \sum_{m \in M} \left( L_m + b_j m_j \sum_{n \in C} a_{n,j} m_n^{\text{em}} \right)$$

$$= \sum_{m \in M} \left( L_m + b_j m_j \sum_{n \in C} \left( H_n + L_n - E_n^{\text{out}} \right) \right).$$ \hspace{1cm} (15)

The recursive expansion of this cost function ends with terminal subsystems in each branch of the tree network. Hence, the complete expansion of (13) is given by

$$\mathcal{F} = \sum_{m \in M} L_m + \sum_{n \in C} \Delta H_n,$$ \hspace{1cm} (16)

which implies that maximization of internal energy is equivalent to minimizing the losses of all subsystems and the internal energy of only the consumers in the network.

These results not only show the importance of pH formulations to obtain physically insightful interpretations of cost functions, but also open the door to obtain possible equivalent cost functions that could be beneficial for specific applications.

In this section, we use this advantage to adapt the Forward-Backward Operator Splitting (FBS) method presented in [13] to find solutions to the pH CVEM problem. The aforementioned FBS method is a parallelizable static optimization technique that takes advantage of the modular structure of the OCP (17).

The FBS method is based on the augmented Lagrangian function

$$\mathcal{L} = F + \sum_{k \in K, m \in M} \sum_{j \in J} \left( x_{m,k} \right) \left( y_{m,k} \right) \left( \lambda_{m,k} \right) \left( \nu_{m,k} \right)$$

$$= \left( x_{m,k} \right) \left( y_{m,k} \right) \left( \lambda_{m,k} \right) \left( \nu_{m,k} \right) \left( x_{m,k} \right) \left( y_{m,k} \right) \left( \lambda_{m,k} \right) \left( \nu_{m,k} \right)$$

$$= \left( \lambda_{m,k} \right) \left( \nu_{m,k} \right) \left( x_{m,k} \right) \left( y_{m,k} \right) \left( \lambda_{m,k} \right) \left( \nu_{m,k} \right) \left( x_{m,k} \right) \left( y_{m,k} \right) \left( \lambda_{m,k} \right) \left( \nu_{m,k} \right)$$

$$= \left( \lambda_{m,k} \right) \left( \nu_{m,k} \right) \left( x_{m,k} \right) \left( y_{m,k} \right) \left( \lambda_{m,k} \right) \left( \nu_{m,k} \right) \left( x_{m,k} \right) \left( y_{m,k} \right) \left( \lambda_{m,k} \right) \left( \nu_{m,k} \right)$$

$$= \left( \lambda_{m,k} \right) \left( \nu_{m,k} \right) \left( x_{m,k} \right) \left( y_{m,k} \right) \left( \lambda_{m,k} \right) \left( \nu_{m,k} \right) \left( x_{m,k} \right) \left( y_{m,k} \right) \left( \lambda_{m,k} \right) \left( \nu_{m,k} \right)$$

$$= \left( \lambda_{m,k} \right) \left( \nu_{m,k} \right) \left( x_{m,k} \right) \left( y_{m,k} \right) \left( \lambda_{m,k} \right) \left( \nu_{m,k} \right) \left( x_{m,k} \right) \left( y_{m,k} \right) \left( \lambda_{m,k} \right) \left( \nu_{m,k} \right)$$

$$= \left( \lambda_{m,k} \right) \left( \nu_{m,k} \right) \left( x_{m,k} \right) \left( y_{m,k} \right) \left( \lambda_{m,k} \right) \left( \nu_{m,k} \right) \left( x_{m,k} \right) \left( y_{m,k} \right) \left( \lambda_{m,k} \right) \left( \nu_{m,k} \right)$$

$$= \left( \lambda_{m,k} \right) \left( \nu_{m,k} \right) \left( x_{m,k} \right) \left( y_{m,k} \right) \left( \lambda_{m,k} \right) \left( \nu_{m,k} \right) \left( x_{m,k} \right) \left( y_{m,k} \right) \left( \lambda_{m,k} \right) \left( \nu_{m,k} \right)$$

$$= \left( \lambda_{m,k} \right) \left( \nu_{m,k} \right) \left( x_{m,k} \right) \left( y_{m,k} \right) \left( \lambda_{m,k} \right) \left( \nu_{m,k} \right) \left( x_{m,k} \right) \left( y_{m,k} \right) \left( \lambda_{m,k} \right) \left( \nu_{m,k} \right)$$

$$= \left( \lambda_{m,k} \right) \left( \nu_{m,k} \right) \left( x_{m,k} \right) \left( y_{m,k} \right) \left( \lambda_{m,k} \right) \left( \nu_{m,k} \right) \left( x_{m,k} \right) \left( y_{m,k} \right) \left( \lambda_{m,k} \right) \left( \nu_{m,k} \right)$$

Note that (17a) is an equivalent formulation of the cost function (16), where \( \epsilon_m = 1 \) if \( m \in C \) and \( \epsilon_m = 0 \) otherwise. This indicates that the cost function (17a) penalizes changes of internal energy only for the set of energy consumers. The discretized versions of internal energy and losses are given respectively by

$$\Delta H_m = \frac{1}{2} \sum_{k \in K} \left( \frac{x_{m,K}^{\text{em}}}{Q_{m,K}} \right)^T Q_{m,K} x_{m,K} - \frac{1}{2} \sum_{j \in J} \sum_{m \in M} a_{j,m} x_{m,j}^{\text{em}}.$$ (18a)

$$\dot{L}_m = \delta_j \sum_{k \in K} \left( x_{m,K} \right) \left( u_{m,k} \right) + \left( x_{m,K} \right) \left( u_{m,k} \right) \left( \lambda_{m,k} \right) \left( \nu_{m,k} \right).$$ (18b)

IV. DISTRIBUTED OPTIMIZATION APPROACH TO pH CVEM

In the previous section we proposed a complete framework to formulate OCP for CVEM applications using pH models. An advantage of this approach is that it is possible to formulate decomposable problems by expanding the cost function in terms of internal energy and losses of the system.

In this section, we use this advantage to adapt the Forward-Backward Operator Splitting (FBS) method presented in [13] to find solutions to the pH CVEM problem (17). The aforementioned FBS method is a parallelizable static optimization technique that takes advantage of the modular structure of the OCP (17).

The FBS method is based on the augmented Lagrangian function

$$\mathcal{L} = f + \sum_{k \in K, m \in M} \sum_{j \in J} \left( x_{m,j} \right) \left( y_{m,j} \right) \left( \lambda_{m,j} \right) \left( \nu_{m,j} \right)$$

where the dependencies in functions \( \mathcal{L} \) and \( F \) have been omitted to improve readability. In (19), \( t_2(z) \) is an indicator function such that \( t_2(z) = 0 \) if \( z \in Z \) and \( t_2(z) = +\infty \) otherwise. Furthermore, we define

$$F \left( \left\{ x_{m,j} \right\}, \left\{ y_{m,j} \right\}, \left\{ \lambda_{m,j} \right\}, \left\{ \nu_{m,j} \right\}, \left\{ \kappa_{m,j} \right\} \right) = \left\{ \left( x_{m,j} \right), \left( y_{m,j} \right), \left( \lambda_{m,j} \right), \left( \nu_{m,j} \right), \left( \kappa_{m,j} \right) \right\}$$

which subject to the discrete-time dynamical model

$$x_{m,k+1} = \left( I + \delta \left( J_m - R_m \right) \right) Q_{m,k} x_{m,k} + \delta B_{m,k} u_{m,k},$$ (17b)

$$y_{m,k} = \left( B_{m,k} + 2P_{m,k} \right) Q_{m,k} x_{m,k} + \left( M_{m,K} + S_{m,k} \right) u_{m,k},$$ (17c)

and discrete-time versions of (10), (11), and (12) where

$$\Omega_m := \left\{ x_{m,k} \in \mathbb{R}^n, y_{m,k} \in \mathbb{R}^n, u_{m,k} \in \mathbb{R}^p \right\}$$

subject to (12b), (12c) and (12d),

$$\left\{ x_{m,k} \right\}, \left\{ y_{m,k} \right\}, \left\{ \lambda_{m,k} \right\}, \left\{ \nu_{m,k} \right\}, \left\{ \kappa_{m,k} \right\}$$

represent dual variables related to the discrete-time versions of the
by defining the following vectors

\[ \mathbf{x}_m = \begin{bmatrix} x_1 \ldots x_K \end{bmatrix}^T \in \mathbb{R}^{nK} \]  

(21a)

\[ \mathbf{u}_m = \begin{bmatrix} u_0 \ldots u_{K-1} \end{bmatrix}^T \in \mathbb{R}^{2K} \]  

(21b)

\[ \mathbf{y}_m = \begin{bmatrix} y_1 \ldots y_K \end{bmatrix}^T \in \mathbb{R}^{2K} \]  

(21c)

\[ \lambda_j = \begin{bmatrix} \lambda_{j,1} \ldots \lambda_{j,K} \end{bmatrix}^T \in \mathbb{R}^{2K} \]  

(21d)

\[ \mu_j = \begin{bmatrix} \mu_{j,1}^T \ldots \mu_{j,K}^T \end{bmatrix} \in \mathbb{R}^{2K} \]  

(21e)

\[ \kappa_j = \begin{bmatrix} \kappa_{j,1}^T \ldots \kappa_{j,K}^T \end{bmatrix} \in \mathbb{R}^{K} \]  

(21f)

\[ \omega_m = \begin{bmatrix} \omega_{m}^T \end{bmatrix} \in \mathbb{R}^{(4+n)K} \]  

(21g)

it is possible to rewrite (20) and (17d) in vector form to obtain \( F(\mathbf{w}_m, \lambda_j, \mu_j, \kappa_j) \) and \( \Omega_m(\mathbf{w}_m) \) respectively. Thus, by applying the operator splitting method presented [13], we can obtain the following static optimization algorithm to find solutions to the pH CVEM OCP (17).

**ALGORITHM 1: Forward-Backward Splitting**

**Initialization:**
Select step-sizes \( \gamma_m \), \( m \in M \), \( \phi_j \), \( j \in J_m \), \( \theta_j \), \( j \in J_n \), and \( v_j \), \( j \in J_p \); select regularization weights \( \eta_j \), \( j \in J_m \), \( \rho_j \), \( j \in J_n \), \( \sigma_j \), \( j \in J_p \) and select initial values \( w_{m0} \in \Omega_m \) for all \( m \in M \), \( \lambda_{j,0} \in \mathbb{R}^{nK} \) for all \( j \in J_m \), \( \mu_{j,0} \in \mathbb{R}^{2K} \) for all \( j \in J_n \), and \( \kappa_{j,0} \in \mathbb{R}^{K} \) for all \( j \in J_p \), and all the iterations \( i \in \mathbb{N} \).

**Iterate until convergence:**

\[ w_{m,i+1} = \arg\min_{w_m} \left\{ \langle w_m - \gamma_m \nabla w_m, F \rangle \right\} \]  

for all \( m \in M \)

\[ \lambda_{j,i+1} = \lambda_j + \phi_j \nabla \lambda_j \]  

for all \( j \in J_m \)

\[ \mu_{j,i+1} = \mu_j + \theta_j \nabla \mu_j \]  

for all \( j \in J_n \)

\[ \kappa_{j,i+1} = \kappa_j + v_j \nabla \kappa_j \]  

for all \( j \in J_p \)

The superscript \( i \in \{1,2,\ldots\} \) denotes the \( i \)-th iteration, the gradient of a continuously differentiable function \( G(\cdot) \) with respect the variable \( z \) is represented by \( \nabla z G(z) \), and the mapping \( \text{proj}_Z : \mathbb{R}^m \rightarrow Z \) for a closed set \( Z \subseteq \mathbb{R}^m \) defines the projection onto \( Z \), i.e., \( \text{proj}_Z(z) = \text{argmin}_{u \in Z} \| u - z \|^2 \).

Algorithm 1 takes advantage of the modularity of CVEM problem by the use of parallel projections that can be efficiently solved using linearly constrained least-squares solvers. Additionally, note that the procedure to obtain Algorithm 1 has been briefly described in this section as a derivation of the work presented in [13]. Although it is possible to rigorously prove that the solutions provided by Algorithm 1 corresponds to solutions of the OCP (12) by following steps similar to the ones described in [13], we consider that this discussion is outside of the scope of this paper and we will leave it for future extensions of this work.

**V. CASE STUDY: BATTERY ELECTRIC VEHICLE**

The pH CVEM framework described in Section II and the associated OCP of Section III are illustrated in a case study presented in this section. We consider a Battery Electric Vehicle (BEV), whose topology is depicted in Fig. 3. A high-voltage battery (HVB) is connected via a DC/DC converter to an electric machine (EM). The mechanical part in this driveline yields a power balance between the requested propulsion power and the mechanical braking power. Even though this configuration is simple, it allows to illustrate the proposed modelling approach for CVEM and the solution method.

In the remainder of this section, we provide details for modelling the topology presented in Fig. 3 and formulate an OCP for CVEM. Later, we discuss the connections between power-based and pH OCP formulations highlighting the advantages of the the approach proposed in this paper, finally, results of simulations for this case study are presented and analyzed.

**A. Modelling**

In the topology presented in Fig. 3 is possible to observe the physical conjugated variables that describe the power preserving interconnection between subsystems. We will use these variables to define the input-output ports for each subsystem. Additionally, the pH models for the subsystems will be obtained from first-principle models and described in terms of the pH representation given in Section II-A.

The set \( M \) is the set of interconnected subsystems for the BEV considered in this case. For the sake of readability, we use \( M = \{ vs, ec, em, br, req \} \) instead of an enumerated set to denote battery voltage source, battery electric circuit, electric machine, mechanical brake and power request, respectively. Note that \( vs \) is a source subsystem, while \( br \) and
req are terminal subsystems. Similarly, $J_n = \{1\}$ represents the mechanical node in the topology, while $J_p = \{2\}$ is the power junction of the network which corresponds to the DC/DC converter.

1) High-Voltage Battery: We consider an equivalent circuit model with constant open circuit voltage $V_{oc}$ and internal resistance $R_{oc}$, in this paper. The voltage and current in the terminals are denoted as $V_{hvb}$ and $I_{hvb}$ respectively. The pH formulation of the HVB consisted of subsystems $vs$ and $ec$, which represent an ideal lossless voltage source and the internal resistance of the battery, respectively. The input and output ports and the states of the aforementioned subsystems are defined as

$$u_{vs} = \begin{bmatrix} 0 \\ -I_{hvb} \end{bmatrix}, \quad y_{vs} = \begin{bmatrix} 0 \\ V_{hvb} \end{bmatrix},$$

(22a)

$$u_{ec} = \begin{bmatrix} V_{hvb} \\ I_{hvb} \end{bmatrix}, \quad x_{ec} = \text{SoC}, \quad y_{ec} = \begin{bmatrix} I_{hvb} \\ -V_{hvb} \end{bmatrix},$$

(22b)

where SoC denotes the HVB state-of-charge. Note that these definitions indicate that the interconnection between the voltage source $vs$ and the electric circuit $ec$ is power preserving, i.e., $y_{ec}^T u_{vs} + y_{ec}^T u_{ec} = 0$. Therefore, the pH models of the subsystems in the HVB are given by

$$Q_{ec} = J_{ec} = R_{ec} = 0, \quad P_{ec} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad B_{ec} = \begin{bmatrix} 0 & -\frac{1}{q_{ec}} \\ \frac{q_{ec}}{R_{ec}} & 0 \end{bmatrix},$$

(22c)

where $q_{hvb}$ denotes the HVB capacity. Numerical values for the parameters associated to the HVB are given in Table I.

2) Electric Machine: The HVB is connected to an EM through an ideal DC/DC converter. The DC/DC converter is considered a power junction in this approach, whose description will be provided later in this section. The pH formulation for the EM is defined using

$$u_{em} = \begin{bmatrix} V_{em} \\ F_{em} \end{bmatrix}, \quad x_{em} = \begin{bmatrix} L_{Lm} I_{Lm} \\ \frac{1}{g} I_{Lm} V_{em} \end{bmatrix}, \quad y_{em} = \begin{bmatrix} I_{em} \\ -V_{em} \end{bmatrix},$$

(23a)

where $V_{em}, I_{em}$ denotes the voltage and current in the EM respectively, $F_{em}$ is the propulsion force and $v_{em}$ the longitudinal velocity. Additionally, the positive constants $L$, $I_{Lm}$, $r_w$ and $g$, denote the electric inductance, moment of inertia, wheel radius and total gear ratio, respectively. These definitions allow the pH model of the EM to be defined using

TABLE I: High Voltage Battery parameters

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Definition</th>
<th>Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{oc}$</td>
<td>688</td>
<td>open circuit voltage</td>
<td>[V]</td>
</tr>
<tr>
<td>$R_{oc}$</td>
<td>0.33</td>
<td>series resistance</td>
<td>[Ω]</td>
</tr>
<tr>
<td>$q_{oc}$</td>
<td>180</td>
<td>capacity of the battery</td>
<td>[Ah]</td>
</tr>
<tr>
<td>$I_{hvb}$</td>
<td>-800</td>
<td>minimum current</td>
<td>[A]</td>
</tr>
<tr>
<td>$I_{hvb}$</td>
<td>800</td>
<td>maximum current</td>
<td>[A]</td>
</tr>
<tr>
<td>$V_{hvb}$</td>
<td>400</td>
<td>minimum voltage</td>
<td>[V]</td>
</tr>
<tr>
<td>$V_{hvb}$</td>
<td>1000</td>
<td>maximum voltage</td>
<td>[V]</td>
</tr>
</tbody>
</table>

TABLE II: Driveline parameters

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Definition</th>
<th>Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{em}$</td>
<td>1</td>
<td>equivalent resistance</td>
<td>[Ω]</td>
</tr>
<tr>
<td>$L$</td>
<td>2</td>
<td>equivalent inductance</td>
<td>[H]</td>
</tr>
<tr>
<td>$I_e$</td>
<td>99.68</td>
<td>equivalent axle inertia</td>
<td>[kg m²]</td>
</tr>
<tr>
<td>$\beta$</td>
<td>1.1e-03</td>
<td>friction coefficient (rotational)</td>
<td>[Nm/s/rad]</td>
</tr>
<tr>
<td>$\kappa_r$</td>
<td>4.2175</td>
<td>torque constant</td>
<td>[Nm/A]</td>
</tr>
<tr>
<td>$\kappa_v$</td>
<td>4.27</td>
<td>back-electromotive constant</td>
<td>[V/s/rad]</td>
</tr>
<tr>
<td>$r_w$</td>
<td>0.5715</td>
<td>wheel radius</td>
<td>[m]</td>
</tr>
<tr>
<td>$g_v$</td>
<td>6.1</td>
<td>gear ratio</td>
<td>[-]</td>
</tr>
<tr>
<td>$I_{em}$</td>
<td>-600</td>
<td>minimum current</td>
<td>[A]</td>
</tr>
<tr>
<td>$I_{em}$</td>
<td>600</td>
<td>maximum current</td>
<td>[A]</td>
</tr>
<tr>
<td>$V_{em}$</td>
<td>-1000</td>
<td>minimum voltage</td>
<td>[V]</td>
</tr>
<tr>
<td>$V_{em}$</td>
<td>1000</td>
<td>maximum voltage</td>
<td>[V]</td>
</tr>
</tbody>
</table>

where $R_{em}$, $\kappa_r$, $\kappa_v$, and $\beta$ are positive constants that denote the equivalent electric resistance, back-electromotive constant, torque constant and damping coefficient, respectively.

In Table II, numerical values of the driveline parameters used for this case study.

3) Mechanical Braking: The function of this subsystem is to dissipate all the power provided to it. It is considered as a terminal subsystem and its ports are defined as

$$u_{br} = \begin{bmatrix} v_{br} \\ 0 \end{bmatrix}, \quad y_{br} = \begin{bmatrix} F_{br} \\ 0 \end{bmatrix},$$

(24)

where $F_{br} \geq 0$ is the mechanical braking force and $v_{br}$ denotes the longitudinal velocity of the vehicle. Note, that a complete description of the subsystem is not necessary because we are only interested in its consumption. Thus, the terms that describe internal energy and losses for the mechanical braking subsystem in the cost function (16) can be replaced by the total energy consumed by the subsystem, i.e.,

$$\Delta H_{br} + L_{br} = \int_{t_0}^{t_1} y_{br}^T u_{br} dt.$$

(25)

4) Power Request: This subsystem describes the power required by the vehicle to travel a certain period of time with a given longitudinal velocity profile $v_{req}$ and traction force $F_{req}$. It is considered a terminal subsystem, whose ports are defined as

$$u_{req} = \begin{bmatrix} v_{req} \\ 0 \end{bmatrix}, \quad y_{req} = \begin{bmatrix} F_{req} \\ 0 \end{bmatrix}.$$

(26)

The requested force and velocity profiles used in this case study are shown in Fig. 4. In this case, a complete description of the subsystem is not provided because its behavior is already given. However, to preserve consistency in the cost function formulation (16), the terms that describe internal
energy and losses are replaced by the total power consumed by the subsystem, i.e.,
\[
\Delta H_{req} + L_{req} = \int_{t_i}^{t_f} y_{req} u_{req} dt.
\] (27)

5) Interconnections: The set \( \mathcal{J} = \mathcal{J}_s \cup \mathcal{J}_r = \{1,2\} \) contains the two junctions in network topology depicted in Fig. 3. This junctions are described by
\[
b_{\text{em},1} = a_{\text{em},1} = a_{\text{req},1} = 1, \quad (28a)
b_{\text{em},2} = a_{\text{em},2} = 1, \quad (28b)
\]
and zero for the remaining coefficients.

B. Optimal Control Problem

After describing pH representations for all the driveline components and its interconnections, the procedure to formulate a CVEM OCP for this case study is reduced to a simple substitution. In particular, a continuous-time CVEM OCP for this case study is obtained by substituting (22)-(28) into (12) and (16). Similarly, substituting (22)-(28) into (17) and (18) generates a discrete-time OCP.

Interestingly, for this case study it is also possible to find and equivalent OCP using a power-based approach. For instance, let us consider the modelling framework proposed in [10]. In this approach, the source subsystem is represented as linear system that describes the accumulation of energy and the conversion of power in all subsystems is defined by a quadratic function of the form
\[
P_{\text{out}} = \frac{1}{2} \gamma_2 P_{\text{in}}^2 + \gamma_1 P_{\text{in}} + \gamma_0,
\] (29)
where \( P_{\text{out}}, P_{\text{in}} \) represent the power in the input and output port of the subsystem, respectively. To illustrate the connection between the power-based approach in [10] with the pH framework proposed in this paper. Let us consider the EM model described by (2) and (23) in steady state. After some algebraic manipulations and by defining \( P_{\text{out}} = V_{\text{em}} I_{\text{em}} \) and \( P_{\text{in}} = F_{\text{em}} V_{\text{em}} \), it is possible to obtain
\[
\gamma_2 = \frac{R_e}{\kappa^2_v}, \quad \gamma_1 = \frac{R_e \beta}{\kappa^2_v}, \quad \gamma_0 = \frac{R_e \beta^2 v_{\text{em}}}{\kappa^2_v} + \frac{\kappa v_{\text{em}}}{\kappa_v}.
\] (30)

Hence, considering that the longitudinal velocity \( v_{\text{em}} \) is given, equation (29) describes only a static relation between input and output powers. Performing a similar procedure in the rest of the subsystems of this example, it allows us to obtain an equivalent OCP that has power in the input-output ports of the subsystems as decision variables.

The advantage of this equivalent formulation is that if static optimization methods are used to obtain solutions, the number of decision variables is lower than in the port-based case. However, this framework fails to describe problems of higher complexity. For instance, see [14] where an attempt to unify the optimization of velocity profiles, i.e., eco-driving, and CVEM for the subsystems of a vehicle is presented. Using the pH framework for CVEM presented in this paper the integration of eco-driving and CVEM is natural. In fact, it is equivalent to include the longitudinal vehicle dynamics as an additional subsystem in the port-based network.

C. Simulation Results

We use Algorithm 1 to find a solution to the discrete-time OCP obtained from the substitution of (22)-(28) into (17) and (18). In this case, we use a sampling time \( \delta_t = 1[s] \) and a horizon of length \( K = 990 \). The results obtained are compared to a benchmark case where all the mechanical braking power is zero, which means that the energy produced by deceleration of the vehicle is completely stored in the HVB. In Fig. 5, the SoC and the mechanical braking force can be observed as function of time for both the benchmark as well as the optimal solution. The final SoC corresponding to the optimal solution is 0.0717% higher than the benchmark solution, which implies energy savings of 4.896% for the given power request. Interestingly, the optimal solution still requires some mechanical braking which clearly shows that the optimal solution selects efficient operational points for the subsystems instead of only trying to recover as much mechanical energy as possible.

Physical insight about the HVB operation for both cases is given Fig. 7 and Fig. 6, where the behavior of battery voltage and current, respectively, is depicted for both solutions. Note that regions where the voltage in the battery terminals increases correspond to negative currents in the HVB. This implies charging the battery, which is also observed in Fig. 5. An advantage of the pH CVEM framework proposed in this paper is that is possible to give independent treatment to each conjugated variable in the port of the subsystems, i.e., setting independent constraints to each variable. For
this study case, the constrains on the voltage and current of HVB aim to capture the charge acceptance phenomenon in the battery. This is not directly possible in power-based approaches because voltage and current are combined as in the battery. This is not directly possible in power-based approaches for CVEM that has allowed us to unify the concepts of energy storage and losses directly in the dynamical models of the subsystems. We have highlighted the relevance of the pH framework in terms of formulation of physically insightful decomposable OCP. Moreover, the extra freedom obtained by describing power in terms of conjugated variables has open the door to include additional physical phenomena in the as constraints in the OCP. Finally, we have adapted the distributed optimization algorithm for power-based CVEM proposed in [13] to find solutions to the pH CVEM problem.

REFERENCES

Traffic-Aware Vehicle Energy Management Strategies via Scenario-Based Optimization *

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Abstract: This paper explores the development of traffic-aware energy management strategies by means of scenario-based optimization. This is motivated by the fact that real driving conditions are subject to uncertainty, thereby making the real-time optimization of the energy consumption of a vehicle a challenging problem. In order to deal with this situation, we employ the current framework of complete vehicle energy management in a receding horizon fashion in which we consider random constraints representing realizations of the disturbance, i.e., the uncertain driving conditions. Additionally, we study three methods for velocity prediction in energy management strategies, i.e., a method based on (average) traffic flow information, a method based on Gaussian process regression, and a method that combines both. The proposed strategy is tested with real traffic data using a case study of the power split in a series-hybrid electric vehicle. The behavior of the battery, control inputs and fuel consumption generated from the randomized strategy are compared against the optimal solution from an online benchmark and a situation with perfect prediction of the future, where the use of a Gaussian process regression and the average traffic speed achieves near optimal fuel consumption.

Keywords: Vehicle Energy Management, Model Predictive Control, Scenario Optimization, Power request predictions

1. INTRODUCTION

Nowadays, vehicle efficiency has become more relevant due to the need to mitigate the environmental impact of fossil fuels and meet the CO₂ emission targets set for 2030 as expressed in the ”Global EV Outlook 2018” (International Energy Agency, 2018) and ”IEA New Policies Scenario” for 2030 as expressed in the IEA New Policies Scenario (International Energy Agency, 2018). In fact, several countries are proposing restrictions to phase out the commercialization and use of non-electrified vehicles in the city centers between 2025 and 2040. However, e-mobility faces a crucial problem to experience a total incorporation in the transportation market denominated range anxiety, i.e., users are concerned about not having enough energy to reach their final destination (Melliger et al., 2017). From this perspective, the development and implementation of Energy Management Strategies (EMSs) becomes a relevant research topic in the automotive industry, because its implementation does not require substantial hardware modifications to achieve longer traveling distances using only a reduced amount of energy.

Basically, EMSs determine how to optimize the energy consumption of a vehicle by establishing an appropriate division of the energy used by its components and subsystems, which in a broader sense is referred as Complete Vehicle Energy Management (CVEM) (Kessels et al., 2012). Generally, the EMS literature can be divided into online and offline methods. Offline methods are typically based on Dynamic Programming (Pérez et al., 2006), Pontryagin’s Minimum Principle (Serriu et al., 2009) or static optimization techniques (Khalik et al., 2018; Padilla et al., 2019), and require the drive cycle to be known a priori. These methods are not real-time implementable and neglect the presence of uncertain driving conditions, e.g., traffic congestion, varying speed limits and different driving styles. Alternatively, different online methods explored in the literature are given by rule-based techniques (Jalil et al., 1997), expressing the energy required by the subsystems through Equivalent Consumption Minimization Strategies (ECMS) (Sciarratta et al., 2004) or implementing Model Predictive Control (MPC) methods with online predictions of the driving mission (Romijn et al., 2017).

Despite being online implementable, all these methods required tuning, which often relies on offline solutions, or assume exact predictions of the power request, limiting their use under real-life situations. Alternatively, stochastic optimal control methods provide noticeable extensions for Traffic-aware Energy Management Strategies (TaEMS), i.e., strategies that take into account the uncertainty present in real traffic conditions. These strategies are typically obtained using Stochastic Dynamic Programming (SDP) (Moura et al., 2011; Johansson et al., 2007), which suffers from scalability problems known as ”Curse of Dimensionality”, or Stochastic MPC (Di Cairano et al., 2014), which might become computationally demanding when the number of subsystems considered in the control problem increases.

* This work has received financial support from the Horizon 2020 program of the European Union under the grant ‘Electric Vehicle Enhanced Range, Lifetime And Safety Through INGenious battery management’ (EVERLASTING-721377)
In this paper, we use the recent developments of scenario-based optimization (Campi and Garatti, 2018; Schidlbach et al., 2013, 2014) to extend the current framework of CVEM as in Romijn et al. (2017). This will lead to a tractable method for traffic-aware complete vehicle energy management, in which intuitive tradeoff can be made between computational complexity and robustness depending on the number of scenarios considered. Furthermore, the proposed method has the potential of using distributed optimization techniques to improve its implementation capabilities (although this will not be addressed in this paper). In addition, we propose the use of Gaussian Processes Regression for this TaEMS to generate multiple predictions of the future driving situations, i.e., sample random scenarios, which are combined with traffic flow information to provide long-term speed predictions.

This paper is organized as follows: In Section 2, the general vehicle energy management problem formulation is presented and extended as an uncertain optimal control problem. A description of the prediction methods for traffic-aware vehicle energy management is included in Section 3. Section 4 presents the implementation details and simulation results obtained on a case study. Finally, conclusions and future work are presented in Section 5.

2. TRAFFIC-AWARE VEHICLE ENERGY MANAGEMENT

In this section, we present the mathematical formulation describing the optimal control problem arising from the CVEM framework in a receding horizon fashion. Additionally, an extension of the resulting Receding-Horizon Optimal Control Problem (RHOCP) in the context of scenario-based optimization is introduced to account for uncertain disturbances affecting a vehicle, i.e., the uncertain power request caused by, e.g., unknown driving conditions.

2.1 Receding Horizon Optimal Control Problem

In general, the CVEM problem aims to define the optimal energy flows between the subsystems in the power network of a vehicle over a prediction horizon $k \in \mathcal{K} = \{0, 1, \ldots, K-1\}$ given the measurements at time step $t \in \mathbb{N}$ represented by

$$\min_{\{u_{n,k,t}, y_{m,k,t}\}} \sum_{m \in \mathcal{M}} \sum_{k \in \mathcal{K}} a_{m,k} y_{m,k,t} + b_{m,k} u_{n,k,t}$$

where $u_{n,k,t} \in \mathbb{R}$ are scalar inputs, $y_{m,k,t} \in \mathbb{R}$ are scalar outputs of the converter of subsystem $m \in \mathcal{M} = \{1, \ldots, M\}$, and the coefficients $a_{m,k}, b_{m,k} \in \mathbb{R}$ define the desired cost metric based on the energy consumed by each subsystem at time instant $k + t$. The minimization of (1a) is subject to a set of constraints describing the behavior of the vehicle’s power network and the exchanges of power in it (Fig. 1 shows a general network structure). First, we consider quadratic equality constraints that define the input-output behavior of the converter in each subsystem

$$y_{m,k,t} = \frac{1}{2} \gamma_{m} u_{n,k,t} + \gamma_{m} u_{n,k,t} + \gamma_{m} u_{n,k,t}$$

with $\gamma_{m} \in \mathbb{R}$, $\gamma_{m} \in \mathbb{R}$ and $\gamma_{m} \in \mathbb{R}$ being coefficients that define the efficiency of converter $m \in \mathcal{M}$ predicted at time $k + t$ given information of time $t \in \mathbb{N}$, $k \in \mathcal{K}$.

Furthermore, the network presents different states that are being controlled, imposing constraints based on the linear system dynamics of the energy buffers

$$x_{m,k,t+1} = A_{m,k} y_{m,k,t} + B_{m,k} u_{n,k,t}$$

in which $x_{m,k,t} \in \mathbb{R}^{m}$ and $u_{n,k,t} \in \mathbb{R}$ denote the predicted state and input, respectively, of subsystem $m \in \mathcal{M}$ at time $k + t$, given information at time $t \in \mathbb{N}$ and $k \in \mathcal{K}$. The admissible states and inputs are subject to constraints, i.e.,

$$x_{m,k,t} \in \mathcal{X}_{m} \quad \text{and} \quad u_{n,k,t} \in \mathcal{U}_{m}$$

Moreover, the interconnections of subsystems are described by $\mathcal{J} = \{1, \ldots, J\}$ nodes and no direct interactions between them are considered, i.e., each subsystem can be connected only to a node, resulting in the power balances

$$d_{j}^{f}(y_{m,k,t}, u_{n,k,t}, w_{j,k,t}) \leq 0$$

with

$$d_{j}^{f}(y_{m,k,t}, u_{n,k,t}, w_{j,k,t}) = \sum_{m \in \mathcal{M}} C_{j,m} y_{m,k,t} + d_{j,m} u_{n,k,t} + w_{j,k,t}$$

for all $j \in \mathcal{J}$, $k \in \mathcal{K}$ and $t \in \mathbb{N}$.

In (1e), $w_{j,k,t}$ are disturbances acting on each node, e.g., the power request from the driver or the auxiliaries. Generally for EMS, it is assumed that these disturbances are known in advance or can be perfectly predicted. However, this might not be always true, as they are generated by the environment or external factors, e.g., the driver. Therefore, we can consider that these unknown disturbances have a stochastic nature, turning the CVEM problem (1) into a Stochastic RHOCP. Even though different methods can be used to solve this problem, we make use of the scenario approach (Campi and Garatti, 2018) to solve the resulting uncertain RHOCP in a computationally advantageous way as presented in the remainder of this section.

2.2 Stochastic RHOCP

Before presenting the scenario approach, let us consider a stochastic extensions to problem (1) accounting for the unknown disturbances in nodes $j \in \mathcal{J}$. In particular, we can post the resulting CVEM problem as the following chance-constrained RHOCP

$$\min_{\{u_{n,k,t}, y_{m,k,t}\}} \sum_{m \in \mathcal{M}} \sum_{k \in \mathcal{K}} a_{m,k} y_{m,k,t} + b_{m,k} u_{n,k,t}$$

subject to (1b) - (1d), and

$$Pr\{d_{j}^{f}(y_{m,k,t}, u_{n,k,t}, w_{j,k,t}) \leq 0\} \geq 1 - \epsilon_{j}$$

where the parameters $\epsilon_{j}$ are acceptable infeasibility levels and the functions $g_{j}$ are defined as in (1e).
The main restriction in (2) is the need to guarantee that the chance-constraints (2b) will hold for any realization of the uncertain disturbances \( w_{j,k,t} \), since the distribution of the uncertainty might be unknown and, even if it is known, the solution could lead to a more conservative solution and undesired performance.

2.3 Scenario-Based Traffic-Aware Energy Management

In order to deal with the restrictive characteristics of the chance-constrained formulation in the previous section, we make use of the scenario approach (Campi and Garatti, 2018) instead. This methodology for data-driven optimization aims to solve the resulting chance-constrained ROHCP by means of a deterministic approximation that considers only a finite number of realizations of the disturbances \( w_{j,k,t} \), thereby providing a tuning knob to balance robustness versus performance of the scenario solution. This allows to achieve a computationally tractable problem when multiple subsystems are considered, in comparison to classic stochastic EMS based on SDP (Moura et al., 2011; Johannesson et al., 2007). With this in mind, the power balances (2b) in problem (2) can be replaced by a set of randomly sampled constraints, leading to what we refer to as scenario-based Traffic-aware Energy Management Strategy (TaEMS).

Now, following the scenario approach and the results in (Schödlbach et al., 2013, 2014), we introduce some definitions and assumptions required for the scenario-based ROHCP:

1. The uncertainties \( w_{i,j,k,t}^{[l]} \) of each node are contained in a single variable \( w_{i,j,k,t}^{[l]} = [w_{i,j,k,t}^{[1]}, \ldots, w_{i,j,k,t}^{[L]}] \), which is a random variable with (maybe unknown) probability measure \( Pr \) and support set \( W \).

2. A sequence of variables \( \{w_{i,j,k,t}^{[l]}\}_{l=1}^{L} \) is a realization of the uncertainty \( w_{i,j,k,t}^{[l]} \) over the prediction horizon defining the scenario \( w_{i,j,k,t}^{[l]} \).

3. Enough independent and identically distributed samples \( w_{i,j,k,t}^{[l]} \) can be obtained at every time instant, giving a set of scenarios \( I = \{1, \ldots, I\} \).

4. The scenario-based ROHCP problem has a feasible solution for almost any \( w_{i,j,k,t}^{[l]} \).

With these definitions, the resulting scenario-based TaEMS problem is given by

\[
\min \sum_{k \in K} a_{m,k} y_{m,k,t} + b_{m,k} u_{m,k,t} \quad (3a)
\]

subject to

\[
y_{m,k,t} = \frac{1}{2m} u_{m,k,t}^2 + \gamma_1 u_{m,k,t} + \gamma_0 m \quad (3b)
\]

\[
x_{m,k+1,t} = A_{m,k} x_{m,k,t} + B_{m,k} u_{m,k,t} \quad (3c)
\]

\[
x_{m,k,t} \in X_{m,k} \quad \text{and} \quad u_{m,k,t} \in U_{m,k} \quad (3d)
\]

and

\[
g_j(y_{m,k,t}, u_{m,k,t}, w_{i,j,k,t}^{[l]}) \leq 0, \quad (3e)
\]

for all \( i \in I \) and \( j \in J \), and \( k \in K, m \in M \) at time \( t \in T \).

From the above formulation, we make use of the results in Schödlbach et al. (2013) to address the selection of the number of scenarios required for a particular feasibility level. To this end, let us define the probability of constraint violation

\[
V_{j,k}(y_{m,k,t}^{*}, u_{m,k,t}^{*}) = Pr\{g_j(y_{m,k,t}^{*}, u_{m,k,t}^{*}, w_{i,j,k,t}^{[l]}) > 0\} \quad (4)
\]

where \( u_{m,k,t}^{*}, y_{m,k,t}^{*} \) refer to the scenario solution. It has been shown in Campi and Garatti (2018) and Schödlbach et al. (2013) that \( V_{j,k}(y_{m,k,t}^{*}, u_{m,k,t}^{*}) \) is bounded by a Beta distribution \( B(\rho_j, I - \rho_j + 1) \), such that

\[
Pr^I\{V_{j,k}(y_{m,k,t}^{*}, u_{m,k,t}^{*}) > \epsilon_j\} \leq B(\rho_j, I - \rho_j + 1) \quad (5)
\]

where \( \rho_j \) is the support rank of the constraint in node \( j \). Here, we make use of the results in Schödlbach et al. (2013) instead of considering the number of decision variables as in the classic scenario approach presented in Campi and Garatti (2018). This is favorable as only a reduced number of the decision variables in problem (3) is affected by (3e) at every step on the prediction horizon \( h \) regardless of the number of sampled scenarios.

From this formulation, we aim to find the minimum number of samples required to satisfy the original chance constraint, as the more samples are drawn, the more conservative the solution becomes. Nevertheless, given that new samples are drawn at each time step, we consider a bound on the expected violation probability \( E\{V_{j,k}(y_{m,k,t}^{*}, u_{m,k,t}^{*})\} \leq \epsilon_j \), which leads to a sample size of \( \epsilon_j \leq \rho_j/(I + 1) \). This result follows from the integration of (5), which can be interpreted as the probability that the \( I + 1 \) sample becomes a support constraint, i.e., the solution obtained with the scenarios \( I \) does not satisfy the power balance \( g_j(y_{m,k,t}^{*}) \leq 0 \) (see Campi and Garatti (2018); Schödlbach et al. (2013, 2014) for further details and proofs).

3. SCENARIO GENERATORS

In order to make predictions of the unknown traffic conditions, and thus the disturbance \( w_{j,k,t} \), we present the three velocity prediction methods used in this work, e.g., predictions based on GPS/eHorizon data, predictions using a Gaussian Process Regression (GPR) model, and a mixed generator that combines both the GPS and the GPR model.

3.1 GPS / eHorizon Methods

First, we consider that the vehicle has access to traffic information through a Global Positioning System (GPS) or an electronic horizon (eHorizon), which are devises available in today’s vehicles. Here, the average traffic speed is calculated based on the traffic flow through a particular section of the road as follows:

\[
u_{m,w}^{[l]} = \frac{z_i}{z_{m,w}} \sum_{j=1}^{Z_{m,w}} v_{n}^{[l]} \quad (6)
\]

where \( z_i \) are the road sections and \( Z_{m,w} \) is number of vehicles passing through a particular road section during a time window, e.g., an update frequency between 1 to 5 minutes, as is usually done in mapping and traffic management systems (Ahmed-Al-Sobky and Mossa, 2015).
3.2 Gaussian Process Regression

Since the average traffic speed only provides a deterministic estimate of the traffic situation, a probabilistic model to forecast the future speed of the vehicle is proposed in this section. In particular, we propose to use a Gaussian Process Regression (GPR) model (Rasmussen and Williams, 2006). The selection of this non-parametric model is motivated by the remarkable prediction capabilities achieved with machine learning methods in (Sun et al., 2015; Lefèvre et al., 2014; Liu et al., 2019) and, at the same time, its particular ability to provide a direct measure for the uncertainty of the predictions. For our application, the GPR model is defined as a Nonlinear Auto-Regression Model (NAR-GP), which results in regressing a function $y_k = f(x_k) + e_k$ that maps vector inputs $x_k \in \mathbb{R}^d$ to scalar outputs $y_k \in \mathbb{R}$, where we feed a feature vector $x_k = \{v_{k-1}, \ldots, v_{k-|\mathcal{I}|}\}$ and predict the future speed $y_k = \{v_{k+1}\}$. Additionally, $e_k \sim \mathcal{N}(0, \sigma^2)$ is a noise term acting on the output of the function and $f \sim \mathcal{GP}(\mu, \ker)$, which is a GPR defined with a prior distribution with $\mu = 0$, a covariance/kernel function $\ker$, e.g., squared exponential, Matérn, rational quadratic exponential, etc., and a set of hyper-parameters $\Theta$. Furthermore, the definition of these hyper-parameters is done by minimizing the negative log-likelihood function on training data $D = \{(x_n, y_n) \mid n = \{1, \ldots, N\}\}$, see (Rasmussen and Williams, 2006) for further details.

Once the NAR-GP is fully defined, we obtain a model that allows us to generate random samples from the posterior distribution, such that

$$v_{k+1|t} \sim \mathcal{P}(y_k|D, x_k) = \mathcal{N}(f_{\text{post}}, \ker_{\text{post}} + \sigma^2; \Theta) \quad (7a)$$

in which

$$f_{\text{post}} = \ker(x, x)\ker + \sigma^2\mathbf{I}^{-1}y \quad (7b)$$

$$\ker_{\text{post}} = \ker(x, x) - \ker(x, x)\ker + \sigma^2\mathbf{I}^{-1}\ker(x, x) \quad (7c)$$

with $y$ the output training data, $\mathbf{I}$ the identity matrix and points $x$, $y$, referring to a test input and output, respectively.

Although predicting over multiple time steps ahead with the NAR-GP results in predictions at uncertain inputs, i.e., regressing the prediction of $v_{k+1|t}$ that is a random variable, we only consider a naive approach to generate the predictions over the prediction horizon $K$, since it simplifies the prediction process and avoids large speed changes which might result in intractable predictions.

3.3 Mixed Generator Approach

Given that long prediction horizons lead to a better performance of the EMS (Romijn et al., 2017), a combination of the velocity prediction methods is considered in this work in order to exploit the benefits of each method, e.g., account for the uncertainty in a short-term and preserve the preview of the traffic situation given by the average traffic speed. This combination is motivated by the fact that most of the maneuvers in car following or traffic situations require a very short time (see Lefèvre et al. (2014) and references therein) and the mismatch present between the traffic speed and the individual speed profiles. At the same time, it is known that machine learning methods tend to incur in large prediction errors when the prediction horizon length increases, as these methods are not able to account for long-term traffic dependencies (Liu et al., 2019). In fact, most of the methods present in the literature are restricted to predictions of 10 seconds in the future and, therefore, the integration of external information could lead to substantial fuel savings while generating more robust solutions against the actions of the driver.

4. CASE STUDY

In this section, we first define the case study considered to evaluate the potential of the proposed scenario formulation. The case study is based on TaEMS for a series hybrid vehicle. We will start by presenting the RHOCO formulation, followed by the selection of the sample size and the explanation of the power request determination.

4.1 Receding Horizon Optimal Control Problem

The case study in this paper is based on the series-hybrid electric vehicle (SHEV) presented in (Khalik et al., 2018). In particular, the powertrain topology of the vehicle is represented as the network of energy buffers depicted in Fig. 2. In this figure, EGU stands for Engine Generator Unit, with $y_{\text{eq}}$ being the fuel consumption and $u_{\text{eq}}$ the power supplied by the EGU to the power network. Furthermore, $u_{\text{hvb}}$ and $y_{\text{hvb}}$ define the electric power coming from the High-Voltage Battery (HVB) and $z_{\text{hvb}}$ represents the stored energy in the battery. Besides this subsystems, the Electric Motor (EM) provides $y_{\text{em}}$, which represents the (unknown) power request defined by the driver. Note that $y_{\text{em}}$ propels the vehicle when being a positive magnitude and negative values denote braking actions.

The node interconnecting the elements in the network defines a power balance as in (1e), where the parameters $c_{\text{em}} = -1$, $d_{\text{eq}} = -1$ and $c_{\text{em}} = 1$ are specified according to the flow direction of the power for each subsystem and all the others are set to zero. On top of this, an external braking signal $y_{\text{br}}$ is introduced to account for

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
& EGU & HVB \\
\hline
$\gamma_{\text{egr}}$ & $2 \cdot 10^{-4}$ & 2.52 \\
$\delta_{\text{egr}}$ & 210 [kW] & 0 [kW] \\
\hline
$\gamma_{\text{hvb}}$ & $1.674 \cdot 10^{-2}$ & 1 \\
$\delta_{\text{hvb}}$ & 92.4 [kW] & 11,988 [kJ] \\
\end{tabular}
\caption{Powers system model coefficients}
\end{table}
the mechanical braking dissipating the excess of energy in the powertrain.

According to the problem formulation in Section 2.1, the task of reducing the fuel consumed by the SHEV is described by cost function with $a_{eq,i}k[t] = r_k$ and all the remaining parameters $a_{eq,i} = b_{eq,i} = 0$, as they do not contribute to the objective of the problem. In this case, $y_{em,k[t]}$ is considered to be an uncertain consequence of the driver actions and only the power request $y_{em,i[t]}$ is known at the current time step $t$, which is a realistic assumption given the on-board sensors in today’s vehicles. Furthermore, Table 1 presents the parameters defining the powertrain in this case study.

After some substitutions, the problem can be reformulated as a Quadratically Constrained Quadratic Program (QCQP) which can be efficiently solved with specialized solvers, e.g., CPLEX CPL (2019), resulting in

$$\min_{(u_{m,k}[t])} \sum_{k \in K} \tau_k \left( \frac{1}{2} g y_{eq,i} u_{eq,i}^2 + \gamma_1 y_{eq,i} u_{eq,i} + \gamma_0 y_{eq,i} \right)$$

subject to

$$\frac{1}{2} g y_{eq,i} u_{eq,i}^2 + \gamma_1 y_{eq,i} u_{eq,i} + \gamma_0 y_{eq,i} \geq -y_{em,k[t]}$$

for all $t \in I$ and

$$x_{vkb,k-1[t]} = x_{vkb,k[t]} - \frac{\tau_k y_{vkb,k[t]}}{a}$$

with $k \in K$, $m \in M$ and $t \in T$. Note that this problem is convex due as $\gamma_2 m > 0$, which leads to a direct definition of the parameters for the scenario-based RHOCP.

### 4.2 Sample Size for Scenario-based RHOCP

Since the power balance (8b) is present for every prediction of time $k + t$, given information at time $t$, the problem has K scenario constraints and affect only the particular inputs at that stage in horizon $K$. Therefore, it is straightforward to define the support rank of the scenario constraint and specify the required sample size. In this case, the sample rank of (8b) is given by $\rho = d - L = 2$, leading to an expected violation probability $E^2[V_i(x_{m,k[t]}^j)] \leq \frac{1}{2^\rho}.$

In this case study, we have defined a sample size of $I = 119$ implying a theoretical bound of $\epsilon = 2/120 = 1.66\%$.

### 4.3 Power Request Definition

Given that the scenarios forecast possible velocity profiles, the power request is calculated using a longitudinal vehicle dynamics model, such that:

$$y_{em,k[t]} = \frac{1}{r_k} \left( \frac{v_{k[t]}(s_{k[t]} + \sigma v_{k[t]}^2 + \sigma v + g \sin(\theta (s_{k[t]})))}{r_k} \right)$$

where $\sigma = g c_d, \sigma = \frac{1}{r_k}, \sigma = g c_d A_f$ and $\theta$ define the rolling resistance, inverse of the mass, aerodynamic drag and the road slope, respectively. The definition and values of these coefficients for the vehicle considered in this case study can be found in Table 2. Moreover, a constant road slope $\theta = 0$ is considered based on the driving cycles and available information.

### 4.4 Example of Scenario-based RHOCP

In order to analyse the performance of the scenario generators and the TaEMS for the power split problem, we first describe the particular characteristics considered in the simulations and later, we present results obtained with each method. Here, the solutions of the TaEMS are compared to the optimal performance given by an offline benchmark and an online solution with perfect prediction of the future driving cycle. Additionally, we assess the benefits of hybridization under uncertain predictions by including the fuel consumption generated by a conventional vehicle, where the power request is directly covered by the combustion engine.

For this case study, we used the traffic data set from the Mobile Century field experiment (Herrera et al., 2010), which was a project carried out to evaluate the use of GPS-enabled smartphones for accurate traffic information systems. The data set consists of position, velocity and time information of individual vehicles driving over a 16 km section of the Interstate 880 highway in California from 10 a.m. to 6 p.m., Fig. 3 shows the different vehicle trajectories in the northbound direction (see Herrera et al. (2010) for descriptions). Particularly, we consider different driving cycles recorded between 10 a.m. and 12 noon due to the presence of a traffic congestion event around 10:30 a.m., which allows for a more complete evaluation of the proposed TaEMS. The different driving cycles selected to evaluate the performance of the TaCEV are shown in Fig. 4.

For implementation, we use a moving average filter with a Gaussian window to smooth intractable speed changes present in the predictions due to the piece-wise constant profiles obtained from the traffic speed or large noise realizations in the NAR-GP samples, as this provided similar results compared to other smoothing methods in the literature (Thorsell, 2018), while keeping more information of the NAR-GP samples.

![Fig. 3. Spatio-temporal vehicle trajectories](image)
5.1 GPS / eHorizon Method

In order to replicate the information supplied by a GPS, we divided the road into 100 segments and considered a time window \( t_w = 300 \) seconds according to the update frequency in (Herrera et al., 2010). A density map depicting the traffic speed obtained with (6) is presented in Fig. 5. For implementation purposes, we consider that the vehicle has access to the average speed relative to the current time of the driving cycle, e.g., at 10:43am the information obtained at 10:40am is known and an update is available at 10:45am.

5.2 Gaussian Process Regression

For this probabilistic velocity prediction method, the training data \( \mathbf{D} \) was composed by real driving cycles taken from the Mobile Century data set starting before 10:30 a.m., and the HWFET, LA92 short and EPA standard driving cycles in order to provide more dynamic data to the model. Furthermore, the covariance function used in this work is the Matérn function, see, e.g., (Rasmussen and Williams, 2006). This selection was motivated by the fact that real driving cycles present long braking patterns, which were not properly captured when using a squared exponential function due to its smoothness characteristics. Additionally, the number of lags in the NAR-GP was set to \( p = 5 \), since it was observed that longer dependencies did not provide a substantial improvement of the predictions.

As an example, the mean predictions generated with the NAR-GP for Test Cycle 1 are shown in Fig. 8, where the top plot presents the prediction accuracy with different prediction horizons and the trajectories for 10 seconds of prediction are shown in the bottom plot. As it can be seen, when the predictions are made for short periods of time, e.g., 10 seconds, 95% of the errors are smaller than 1 m/s, which is acceptable for the development of energy management strategies. Nevertheless, these errors present an increasing trend when generating a velocity forecast for longer horizon lengths. Besides this, the NAR-GP occasionally generates wrong braking predictions, e.g., predictions around second 1100, but taking multiple random samples results in a more cautious use of the battery.

5.3 Prediction Horizon Length

In order to establish an appropriate length of the prediction horizon, the fuel consumption obtained with the average traffic speed and the NAR-GP predictions were evaluated. Fig. 6 shows the the fuel consumption with a perfect prediction, the scenario solution with the NAR-GP and the average traffic speed (GPS), considering the offline benchmark as the baseline for three test cycles. From this figure, it can be seen that a longer prediction horizon resulted in a lower fuel consumption since it allows a larger deviation from the final state constraint imposed in the problem. For test cycle 2, the traffic speed captures the fast deceleration accurately, leading to an appropriate use of the battery compared to the NAR-GP, where the battery is mainly used after the braking event starts, see Fig. 7. Regarding the third test cycle, the use of the average traffic speed incurs in a higher fuel consumption after one minute of prediction due to the mismatch between the traffic flow and the actions of the driver. In the same way, the NAR-GP leads to better fuel economy since the braking event in this driving cycle presents a softer deceleration compared to the other tests and sampling different profiles accounts for the fast accelerations at the end of the cycle.
Fig. 8. NAR-GP predictions for test cycle 1. Top: Error of the predicted mean vs. prediction horizon length. Bottom: Predicted mean speed with horizon of 10 seconds.

5.4 Mixed Scenario Generator

As shown before, the NAR-GP is capable of producing accurate predictions when a short horizon is specified, but fails to anticipate longer braking events. For this reason, a horizon length of 10 seconds was selected to generate the speed profiles and to define a reference point to other velocity prediction models studied in the literature. After this predictions are made, we use the average traffic speed to complete the remaining part of the prediction horizon in order to keep the preview of the future traffic conditions and provide more freedom for the usage of the battery.

Finally, since longer horizons have a large impact in the computation time due to the increment of constraints and decision variables in the RHOCPS, we incorporate the variable step-size approach proposed in (Romijn et al., 2017) for the predictions of the average traffic speed since, as indicated by the authors, coarser predictions of the future have minor impacts in the fuel savings while noticeably reducing the computation complexity of the problem. We consider \((\tau_1, \ldots, \tau_K) = (1, 1, 2, 4, 6, 8, 10, 12, 16, 20, 40)\) as the step-size sequence used to generate long-term predictions, where the specific values of the sequence are defined as suggested in (Romijn et al., 2017), such that the total length of the horizon obtained is 120 seconds.

The resulting fuel savings obtained with the mixed scenario generator are reported in Table 3, where we present the fuel consumed with ‘full mixed’ scenario generators (i.e., with variable step sizes, as explained above) and the results when the variable step size is incorporated and only the average traffic speed is considered while using the same sampling time sequence, i.e., ‘variable GPS’. Moreover, the state trajectory and control inputs for Test cycle 2 are shown in Fig. 9.

From Table 3, it can be seen that the proposed EMS presents a slight improvement when the full predictions of the mixed scenario generator are considered in comparison to the GPS-based predictions (see also Fig. 6). Nevertheless, the main advantage is observed when the decision variables are reduced by means of the variable sampling time \(\tau_1\). In this case, we see that the fuel savings decrease as shown in (Romijn et al., 2017) but the incorporation of multiple predictions keeps the usage of fuel closer to their optimal values and reaching up to 99.3% when full predictions are used and 98.2% with the sampling sequence \(\tau_1\). On the other hand, it can be noticed there exists a negative effect of faulty predictions, as the consumption increases due to the mismatch in the long-term prediction information in Test Cycle 3.

6. CONCLUSIONS

In this paper, a traffic-aware energy management strategy has been developed that accounts for uncertain disturbances. This strategy is based on the solution to a scenario-based optimal control problem that is used in a receding horizon fashion. The scenario-based approach reduces the probability of running out of energy during a driving mission. Different alternatives to include traffic information for the generation of scenarios have been defined and evaluated, where up to 99.27% of the optimality in terms of fuel consumption was achieved with a suitable mix of the available information and 98.24% with variable step-size predictions.

Future work involves the improvement of the scenario generator to increase the accuracy of the predictions while handling more complex driving conditions. Besides, extensions to distributed optimization are suggested by decomposing the problems in terms of subsystems, scenarios and horizon so as to reduce the computational time.

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Vehicle Energy Management with Ecodriving: A Sequential Quadratic Programming Approach with Dual Decomposition

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Abstract—In this paper, we propose to solve the ecodriving problem using a Sequential Quadratic Programming (SQP) algorithm. We formulate the ecodriving problem as discrete-time (possibly nonconvex) nonlinear optimal control problem and solve this using SQP, in which we form a convex SQP subproblem using Tikhonov regularization. We will further show that the SQP algorithm can be embedded in a distributed optimization approach, allowing it to be used for Complete Vehicle Energy Management (CVEM), incorporating optimal control of the vehicles auxiliary systems, in combination with ecodriving. We consider two case studies for the ecodriving problem. The first case study concerns the optimal control of a Full-Electric Vehicle, which has one control input and two states and is solved with the SQP algorithm. The second case study lays a foundation for CVEM with ecodriving, where we will solve an energy management problem with ecodriving for a Series-Hybrid Electric Vehicle, which has three control inputs and three states, using the aforementioned SQP algorithm and dual decomposition.

I. INTRODUCTION

Hybrid electric vehicles offer the potential to reduce fuel consumption of a vehicle by adding an electric motor with a high-voltage battery to the powertrain. This allows braking energy to be recuperated and allows the combustion engine to work at a more efficient operating point. In energy management, supervisory control is used to determine the optimal power flow between the electric machine and the combustion engine. A recent trend is to extend the energy management problem to incorporate more auxiliary devices, such as a refrigerated semitrailer or a climate control system [1], engine thermal management [2], battery thermal management [3], battery ageing [4], [5]. Incorporating all the vehicle energy consumers into the energy management problem is referred to as Complete Vehicle Energy Management (CVEM) in [6]. The rationale behind this is that the auxiliaries also consume a considerable amount of energy. However, in all these vehicle energy management problems, the vehicle speed is assumed to be given, while most of the power generated by the powertrain is used for propelling the vehicle. Therefore, optimizing the speed of a vehicle over a certain trajectory, thereby allowing for an optimal conversion of potential energy from the road profile into kinetic energy of the vehicle, can lead to a considerable energy consumption reduction. In this paper, we refer to this latter problem as the ecodriving problem.

In Vehicle Energy Management (VEM), global optimal solutions are typically achieved using Dynamic Programming (DP) [7], see e.g., [8]. However, DP has the inherent disadvantage that the computational burden increases with the number of states. Optimization methods based on the Pontryagins Maximum Principle (PMP), see, e.g., [4], [9] can handle computational complexity of multi-state energy management problems. In PMP, the problem is reduced to solving a two-point boundary value problem, which can be difficult to solve in the presence of state constraints. Static optimization methods guarantee a global optimal solution for convex approximations of the energy management problems, e.g., [10]. To increase scalability of the static optimization problem to allow for a large number of auxiliary systems, distributed optimization approaches have been proposed in [1] for complete vehicle energy management. Using a dual decomposition, a large optimal control problem is split up into several smaller optimal control problems. A disadvantage of the convex optimization approach to vehicle energy management of [1] is that it requires the powertrain components to be described by (convex) quadratic models. This renders the distributed optimization approach not usable for the ecodriving problem, as the longitudinal vehicle dynamics is nonlinear. However, a particular extension of the distributed optimization approach of [1] towards a nonlinear battery ageing model has been made in [5], showing its ability to handle nonlinear convex models as well.

Some of the above mentioned optimization methods have been applied to the ecodriving problem, e.g., PMP in [11], [12], DP in [13], [14], and static optimization in [15]. The approaches based on PMP and DP suffer from their inherent difficulties, i.e., incorporating state constraints in PMP and more dynamics in DP, while the approach based on (convex) static optimization [15] applies an Euler discretization to a continuous time optimal control problem. As the problem is formulated as a second-order cone program, some of the powertrain components can only be modeled using (piecewise) linear functions. Furthermore, convexity might be lost after applying the Euler discretization.

In this paper, we propose to solve the nonlinear (and possibly nonconvex) ecodriving problem using a static optimization approach. To handle the nonlinearity and nonconvexity of the resulting optimal control problem, we employ a Sequential Quadratic programming (SQP) algorithm [16], [17]. The SQP algorithm is similar in nature to the algorithm in [18], where the hessians are approximated to

This work has received financial support from the Horizon 2020 programme of the European Union under the grant 'Electric Vehicle Enhanced Range, Lifetime And Safety Through INGenious battery management' (EVERLASTING-713771).

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accelerate convergence, however we take advantage of the problem structure and further eliminate the state variables as is often done in MPC. Furthermore, we will show that the presented SQP algorithm can be embedded in the distributed optimization approach of [1], allowing it to be used for complete vehicle energy management, incorporating optimal control of the vehicles auxiliary systems, in combination with ecedriving. We will benchmark our solution strategy on a Full Electric Vehicle (FEV) problem presented in [12], which solves the problem using PMP, and solve the problem using SQP. We show the advantage of SQP over the PMP approach used in [12], as state constraints can be very easily added in our proposed approach. Furthermore, we demonstrate the combined ecedriving problem with the powersplit control of a series hybrid powertrain, as was also considered in [15].

II. ECODRIVING PROBLEM FORMULATION

In this section, we formulate the Ecodriving problem. Moreover, we will propose a discrete-time formulation of the problem and we will show that this leads to a nonconvex optimization problem. We define the Ecodriving problem as minimizing traction power over a certain trajectory:

\[
\min_{v(t), s(t)} \int_{t_0}^{t_f} P_{\text{trac}}(v(t), u(t)) \, dt,
\]

subject to state dynamics,

\[
x_{k+1} = f(x_k, u_k),
\]

and state constraints and input constraints,

\[
x_k \in x_k \in \mathbb{R}^n, \quad u_k \in u_k \in \mathbb{R}^m,
\]

for \( k \in K = \{0, 1, \ldots, K-1\} \), where \( K = \frac{t_f - t_0}{\tau} \) is the optimization horizon with \( x_0, x_K \) given and \( \tau > 0 \), which is chosen such that \( \tau \tau f \in \mathbb{N} \), is the step size. To arrive at a discrete-time approximation of (1), we choose

\[
x_k = \left[ \begin{array}{c} v_k \\ x_k \end{array} \right], \quad H_k = \left[ \begin{array}{ccc} 0 & 0 & \gamma_k \\ \gamma_k & 0 & 0 \\ 0 & \gamma_k & 0 \end{array} \right], \quad F_k = \left[ \begin{array}{c} 0 \\ 0 \end{array} \right],
\]

and

\[
f(x_k, u_k) = \left[ \begin{array}{c} u_k + \tau \left( \sigma_1 v_k - \sigma_2 v_k^2 - \sigma_3 - g \sin(\alpha(s_k)) \right) \\ \sigma_1 v_k + \tau v_k \end{array} \right].
\]

An electric machine, represented by (1b), typically has a close to linear input-output behavior, such that \( \alpha = 1 \), \( \gamma_1 \ll 1 \) and \( \gamma_2 \ll 1 \), as the power losses are small generally. This might cause (2) to be a non-convex optimization problem, as \( H_k \) is note positive definite. However, (2) may still be convex in the feasible domain, i.e., where the constraints (2b) - (2c) are satisfied. We refer to [Paul’s paper] where it is shown that due to the discretization step to arrive at (2), the discretized problem is nonconvex even in the feasible domain for any sampling time \( \tau > 0 \). As a result, the method presented in this paper will give only a local minimum, which might not be the global minimum. A method to arrive at a convex formulation of the ecedriving problem (1) can be found in [Paul’s paper].

III. SQP APPROACH TO ECODRIVING

In this section, we will present a Sequential Quadratic Programming (SQP) algorithm to solve the nonlinear optimal control problem (2). SQP aims at solving a nonlinear optimization problem by sequentially solving linearly constrained quadratic programs (LCQP), which are formed, e.g., by approximating the objective function with a quadratic equation and linearizing the constraints. We will further prove that solving the SQP algorithm yields a solution to (2), provided that the solution converges. In particular, we will solve (2) by recursively solving the SQP subproblem

\[
\{x_k^{i+1}, u_k^{i+1}\}_{k \in K} = \arg \min_{x_k, u_k} \sum_{k \in K} \frac{1}{2} [x_k - x_k]^{T} H_k [x_k - x_k] + F_k [x_k - x_k]^{T} [x_k - x_k],
\]

subject to linearized state dynamics,

\[
x_{k+1} = f(x_k, u_k) + \nabla f(x_k, u_k) [x_k - x_k],
\]

and state constraints and input constraints,

\[
x_k \in x_k \in X, \quad u_k \in u_k \in U,
\]

for all \( k \in K \) and \( i \in N \), and for given \( x_0, x_K \) as well as some suitably well chosen \( \{x_k^0, u_k^0\}_{k \in K} \), such that
a feasible solution exists for the next iteration of the SQP subproblem (4). Thus, we have formed the SQP subproblem (4) by linearizing the state equations (2b), and linearizing the objective function (2a) and adding an $R_k > 0$ to ensure a convex objective function in (4a), which can be regarded as a proximal or Tikhonov regularization. The matrix $R_k$ can be chosen to warrant that the SQP subproblem (4) is strictly convex in its control variables $\{u_k\}_{k \in \mathbb{K}}$ and converges. We note that by choosing $R_k = H_k$, the SQP objective function (4a) becomes exactly the original objective function (2a).

The SQP algorithm can be terminated when, e.g.,

$$|J^{i+1} - J^i| \leq \Delta_{tol},$$

(5a)

in which $\Delta_{tol}$ is a certain specified tolerance, and

$$J^i = \sum_{k \in \mathbb{K}} \frac{1}{2} [x_k^i - \hat{x}_k] \mathbf{H}_k [x_k^i - \hat{x}_k] + F_k [x_k^i - \hat{x}_k] + \nu \sum_{k \in \mathbb{K}} |x_k^{i+1} - f(x_k^i, u_k^i)|$$

(5b)

is the optimal cost at iteration $i$, which can be considered as a merit function for the SQP approach (4). In (5b) $\nu > 0$ is chosen such that infeasible solutions to the SQP subproblem (4) at iteration $i$ return a higher cost than optimal feasible solutions. Note that in this SQP approach, we allow infeasible solutions at iteration $i$, and as the algorithm converges, i.e., $|J^{i+1} - J^i| \to 0$, feasibility is obtained in the limit. Also note that as the state and input constraints (4c) are automatically satisfied with a solution to the SQP (4), and thus there is no contribution from these inequality constraints to the merit function (5b).

We can prove that when the SQP problem has converged, i.e., $x_k^{i+1} = x_k^i$ and $u_k^{i+1} = u_k^i$ for all $k \in \mathbb{K}$, the first-order necessary conditions for optimality, i.e., the so-called Karush-Kuhn-Tucker (KKT) conditions [19], for (4) are identical to the KKT conditions of (2). The KKT conditions use the notion of a Lagrangian, which for (2) is given by,

$$L(x_k, u_k, \lambda_k, \mu_k) = \sum_{k \in \mathbb{K}} \frac{1}{2} [x_k - \hat{x}_k] \mathbf{H}_k [x_k - \hat{x}_k] + F_k [x_k - \hat{x}_k] + \mu_k (x_k^{i+1} - f(x_k, u_k)) + \lambda_k h(x_k, u_k),$$

(6)

where $h(x, u) = (\cdot - x) \cdot (\cdot - u)^\top$. For a local optimal solution $\{x_k^*, u_k^*\}_{k \in \mathbb{K}}$ that satisfy the KKT conditions of the Lagrangian, i.e.,

$$[\mathbf{H}_k x_k^* + F_k + \mu_k^*] - \nabla f(x_k^*, u_k^*)^\top \mu_k^{i+1} + \nabla h(x_k^*, u_k^*)^\top \lambda_k^* = 0,$$

(7a)

and primal feasibility and complementarity slackness of the constraints, i.e.,

$$x_k^{i+1} - f(x_k^i, u_k^i) = 0$$

(7b)

$$0 \leq \lambda_k^* \perp h(x_k^i, u_k^i) \geq 0,$$

(7c)

where the notation $0 \leq a \perp b \geq 0$ indicates $a, b \geq 0$, $a \cdot b = 0$. Similarly, the necessary conditions for optimality of (4) are given by a stationarity condition, i.e.,

$$R_k [x_k - x_k^*] + H_k [x_k - x_k^*] + F_k + [u_k - u_k^*] - \nabla f(x_k^*, u_k^*)^\top \mu_k^{i+1} + \nabla h(x_k^*, u_k^*)^\top \lambda_k^* = 0,$$

(8a)

and primal feasibility and complementary slackness of the constraints, i.e.,

$$x_k^{i+1} - f(x_k^i, u_k^i) + \nabla f(x_k^i, u_k^i) [x_k - x_k^*] + u_k^* \geq 0$$

(8b)

$$0 \leq \lambda_k^* \perp h(x_k^i, u_k^i) \geq 0,$$

(8c)

We observe that when $x_k^* = x_k^{i+1} = x_k^i$ and $u_k^* = u_k^{i+1} = u_k^i$ for all $k \in \mathbb{K}$ at iteration $i \to \infty$, the KKT conditions (8) and (7) are equal. Thus, we can find (local) solutions of (2) by solving the SQP (4), provided that the iterates converge, in the sense that $x_k^{i+1} \to x_k^*$ and $u_k^{i+1} \to u_k^*$ for all $k \in \mathbb{K}$ when $i \to \infty$.

We remark that the state variables may be eliminated in the SQP problem (4), by rewriting the linearized state equations (4b) in a prediction form, where the state variables are given by a set of prediction matrices and the inputs, i.e.,

$$x = \begin{bmatrix} x_0 \\ x_k \\ \vdots \\ x_{K-1} \\ x_{K} \end{bmatrix} = \Phi(x_k^*, u^*) + \Gamma(x_k^*, u^*)u,$$

(9)

where $u = \begin{bmatrix} u_0 \\ \vdots \\ u_{K-1} \end{bmatrix}$ and $\Phi$, $\Gamma$ are some matrices such that (9) represents (4b). By substituting (9) into the objective function (4a) and state constraints (4c), as is often done in model-predictive control, we can arrive at an SQP subproblem with only the input variables as decision variables, and the state variables can be updated with (9).

IV. ENERGY MANAGEMENT WITH ECODRIVING

As a case study for Vehicle Energy Management (VEM) with ecodriving, we consider a series-hybrid electric vehicle, consisting of an electric motor (EM), engine-generator unit (EGU) and a high-voltage battery (HVB). The topology is shown in Fig. 1, in which $y_{egu}$ and $y_{hvb}$ denote the ICE’s fuel and mechanical power, respectively, $y_{egu}$ and $y_{hvb}$ the battery’s electrical and stored chemical power, $y_{hvb}$ is an artificial braking power exerted at the interconnection of the subsystems, $u_{em}$ and $y_{em}$ the EM’s mechanical force and electrical power, respectively. $y_{egu}$ denotes the battery state of energy and $x_{em} = \begin{bmatrix} x_0 \end{bmatrix}$, denotes the states speed $v$ and distance traveled $s$ of the vehicle. In VEM, the main goal is to minimize fuel consumption, given by

$$\sum_{k \in \mathbb{K}} \mathcal{T} \mathcal{U}_{g_{up}}, k,$$

(10a)

Fig. 1: Topology
subject to the dynamics and input-output behavior of the converters in Fig 1, and the power balance at interconnection of the subsystems, i.e.,

$$y_{m,k} - u_{egu,k} - y_{hvb,k} = y_{br,k} \leq 0 \quad (10a)$$

for all $k \in K$. We use the constraint (10b) and energy balance constraint of the HVB, i.e.,

$$\sum_{k \in K} y_{hvb,k} = x_{hvb,K} - x_{hvb,0} = 0 \quad (10c)$$

to rewrite (10a) as a ‘sum of losses’, i.e.,

$$\sum_{k \in K} y_{egu,k} - u_{egu,k} + y_{hvb,k} - y_{br,k} = y_{br,k} \leq 0 \quad (10d)$$

By substituting (10b) and (10c) into (10d), we arrive at the convex SQP problem by formulating it as a convex SQP problem and then apply dual decomposition as presented in [1].

**A. Optimal Control Problem**

The objective is to minimize fuel consumption, for which we use the equivalent fuel consumption (10d), which may be written as

$$\min_{y_{m,k} \in K} \sum_{m \in M} \sum_{k \in K} a_m u_{m,k} + b_m y_{m,k} \quad (11a)$$

where $a_m \in R$ and $y_{m,k} \in R$ are the (scalar) inputs and outputs of the converter in subsystem $m \in M = \{em, egu, hvb\}$ at time instant $k \in K$. The optimization problem (11a) is to be solved subject to an equality constraint that describes the quadratic input-output behavior of each converter, i.e.,

$$y_{m,k} = \frac{1}{2} \left[ \frac{a_m}{u_{m,k}} \right]^T H_m \left[ \frac{a_m}{u_{m,k}} \right] + F_m^T \left[ \frac{a_m}{u_{m,k}} \right] + e_m \quad (11b)$$

for all $k \in K$, $m \in M$, and subject to the system dynamics of the state subsystem in each converter, i.e.,

$$x_{m,k+1} = f_m(x_{m,k}, u_{m,k}) \quad (11c)$$

for all $k \in K$ where the initial state $x_{m,0}$ and final state $x_{m,K}$ of the energy storage device are assumed to be given, and the input $u_{m,k}$ is subject to linear inequality constraints, i.e.,

$$\underline{u}_{m,k} \leq u_{m,k} \leq \overline{u}_{m,k} \quad (11d)$$

for all $k \in K$, $m \in M$ and the subject $x_{m,k}$ is subject to linear inequality constraints, i.e.,

$$\underline{x}_{m,k} \leq x_{m,k} \leq \overline{x}_{m,k} \quad (11e)$$

for all $k \in K$, $m \in M$. Finally, the optimization problem is solved subject to a linear inequality constraint describing the interconnection of the subsystems given by (10b), which we write as

$$\sum_{m \in M} c_m u_{m,k} + d_m y_{m,k} = y_{br,k} \leq 0 \quad (11f)$$

for all $k \in K$, $c_m, d_m \in R$. Note that we have left out the term $-y_{br,k}$ in (11a), as $y_{br,k}$ for all $k \in K$ is already given by the inequality (11f), which renders $y_{br,k}$ as an implicit decision variable. To have that (11a) corresponds to (10d), we choose

$$a_{em} = b_{hvb} = -\tau, \quad a_{hvb} = b_{egu} = b_{em} = \tau, \quad a_{em} = 0 \quad (12a)$$

and to have that (11f) corresponds to (10b), we choose

$$c_{egu} = d_{hvb} = -1, \quad c_{hvb} = c_{em} = d_{egu} = 0 \quad (12b)$$

Finally, we assume in this paper that the coefficients for the input-output behavior of the converters in (11b) are given by

$$H_{em} = \begin{bmatrix} \gamma_{em0} & 0 & 0 \\ 0 & \gamma_{em} & 0 \\ 0 & 0 & \gamma_{em0} \end{bmatrix}, \quad F_{em} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad e_{em} = 0,$$

$$H_{hvb} = \begin{bmatrix} \gamma_{hvb0} & 0 \\ 0 & \gamma_{hvb} \end{bmatrix}, \quad F_{hvb} = \begin{bmatrix} 0 \\ \gamma_{hvb0} \end{bmatrix}, \quad e_{hvb} = \gamma_{hvb}, \quad e_{hvb} = \gamma_{hvb0} \quad (12c)$$

for some parameters $\gamma_{m0}$, $\gamma_{m1}$, $\gamma_{m2}$, $m \in M$, the state dynamics for the EM are given by

$$f(x_{em,k}, u_{em,k}) = x_{hvb,k} - \tau u_{hvb,k} \quad (12d)$$

and the battery state of energy is given by

$$f_{hvb}(x_{hvb,k}, u_{hvb,k}) = x_{hvb,k} - \tau u_{hvb,k} \quad (12e)$$

**B. SQP Formulation**

To form a convex SQP formulation, we propose to relax the (nonconvex) quadratic input-output behavior of the converters (11b) to a convex quadratic approximation by linearizing (11b) around $[\nu_m^k]^T$ and adding a convex quadratic part, i.e.,

$$y_{m,k} \approx \frac{1}{2} \left[ \frac{a_m}{u_{m,k}} \right]^T H_m \left[ \frac{a_m}{u_{m,k}} \right] + F_m^T \left[ \frac{a_m}{u_{m,k}} \right] + e_m, \quad (13)$$

for all $k \in K$ and $m \in M$, where the matrix $R_m \geq 0$ is chosen such that (13) is convex. We note that the approximation error disappears when $x_{m,k} \to \nu_m^k$, $u_{m,k} \to u_{m,k}$, and by choosing $R_m = H_m$, we retrieve the original quadratic equation (11b). By substituting (13) into (11a) and (11f) and linearizing the dynamics (11c), we arrive at the convex SQP subproblem

$$\{x_{m,k+1}^k, u_{m,k+1}^k\} \in K, m \in M = \arg\min_{x_{m,k+1}^k, u_{m,k+1}^k \in K, m \in M} \sum_{m \in M, k \in K} \frac{1}{2} b_m (\frac{a_m}{u_{m,k}} - \frac{a_m^0}{u_{m,k}^0})^T R_m (\frac{a_m}{u_{m,k}} - \frac{a_m^0}{u_{m,k}^0}) + (H_m \frac{a_m}{u_{m,k}} + F_m^T \frac{a_m}{u_{m,k}} + e_m, \quad (14a)$$

subject to the linearized state dynamics, i.e.,

$$x_{m,k+1} = f_m(x_{m,k}, u_{m,k}) - \nabla f_m(x_{m,k}, u_{m,k})(\frac{a_m}{u_{m,k}} - \frac{a_m^0}{u_{m,k}^0}) = 0, \quad (14b)$$

for all $k \in K$, $m \in M$, and subject to linear inequality constraints, i.e.,

$$\underline{x}_{m,k} \leq x_{m,k} \leq \overline{x}_{m,k}, \quad \underline{u}_{m,k} \leq u_{m,k} \leq \overline{u}_{m,k} \quad (14c)$$
for all \( k \in \mathcal{K}, m \in \mathcal{M} \), and further subject to the convex quadratic inequality constraint specifying the interconnection of the subsystems
\[
\sum_{m \in \mathcal{M}} \frac{1}{2} d_m \left( \begin{bmatrix} x_{m,k}^* \\ u_{m,k}^* \end{bmatrix} - \begin{bmatrix} x_{m,k} \\ u_{m,k} \end{bmatrix} \right)^T R_m \left( \begin{bmatrix} x_{m,k}^* \\ u_{m,k}^* \end{bmatrix} - \begin{bmatrix} x_{m,k} \\ u_{m,k} \end{bmatrix} \right) + \left( d_m R_{m,k} \begin{bmatrix} x_{m,k}^* \\ u_{m,k}^* \end{bmatrix} + d_m F_{m,k} + \left( \begin{bmatrix} u_{m,k}^* \\ \nu_{m,k} \end{bmatrix} \right)^T \begin{bmatrix} u_{m,k} \\ \nu_{m,k} \end{bmatrix} + d_m e_m \right) \leq 0,
\]
(14d) for all \( m \in \mathcal{M} \).

C. Dual Decomposition

Note that (14a) subject to (14b) and (14c) is entirely separable, and the only complicating constraint is (14d), which is the constraint that acts on all components \( m \in \mathcal{M} \). Therefore, we propose to decompose (14) via dual decomposition by augmenting the objective function with the constraint (14d), which results in the so-called 'partial Lagrangian':
\[
L(\{x_{m,k}, u_{m,k}, \lambda_k\}) = \sum_{k \in \mathcal{K}, m \in \mathcal{M}} \frac{1}{2} \begin{bmatrix} \nu_{m,k}^* \\ u_{m,k}^* \end{bmatrix}^T R_{m,k} \begin{bmatrix} \nu_{m,k} \\ u_{m,k} \end{bmatrix} + \left( \begin{bmatrix} \nu_{m,k}^* \\ \lambda_k \end{bmatrix} \right)^T \begin{bmatrix} \nu_{m,k} \\ \lambda_k \end{bmatrix} + \tilde{E}_{m,k},
\]
(15a)
in which,
\[
\tilde{R}_{m,k} = (b_m + d_m \lambda_k) R_m, \quad \tilde{H}_{m,k} = (b_m + d_m \lambda_k) H_m, \quad \tilde{F}_{m,k} = (b_m + d_m \lambda_k) F_m + (a_m + c_m \lambda_k) \nu_k, \\
\tilde{E}_{m,k} = (b_m + d_m \lambda_k) e_m,
\]
(15b)
where \( \lambda_k \in \mathbb{R}^N \) is a Lagrange multiplier. The partial Lagrange dual function is then given by
\[
g(\{\lambda_k\}) = \min_{x_{m,k}, u_{m,k}} L(\{x_{m,k}, u_{m,k}, \lambda_k\}) = \sum_{m \in \mathcal{M}} g_m(\{\lambda_k\}) + \tilde{E}_{m,k},
\]
(16a)
subject to (14b) and (14c), with
\[
g_m(\{\lambda_k\}) = \min_{x_{m,k}, u_{m,k}} \frac{1}{2} \begin{bmatrix} \nu_{m,k}^* \\ u_{m,k}^* \end{bmatrix}^T R_{m,k} \begin{bmatrix} \nu_{m,k} \\ u_{m,k} \end{bmatrix} + \left( \begin{bmatrix} \nu_{m,k}^* \\ \lambda_k \end{bmatrix} \right)^T \begin{bmatrix} \nu_{m,k} \\ \lambda_k \end{bmatrix} + \tilde{H}_{m,k} \begin{bmatrix} \nu_{m,k}^* \\ u_{m,k}^* \end{bmatrix} + \tilde{F}_{m,k} \begin{bmatrix} \nu_{m,k} \\ \lambda_k \end{bmatrix}^T \begin{bmatrix} u_{m,k} \\ \nu_{m,k} \end{bmatrix} + \tilde{E}_{m,k},
\]
(16b)
subject to (14b) and (14c), the dual problem for each component. Note that for the components whose dynamics \( f_m(x_{m,k}, u_{m,k}) \) are linear and \( R_m = H_m \) yields a convex dual problem (16b), SQP is not needed to solve the dual problem, as they can be solved as a QP problem. Typically, \( g_{\text{SQP}} \) and \( g_{\text{DQP}} \) are both QP problems and \( d_m \) is an SQP problem that can be solved through the approach given in Section III. However, it may still be beneficial to apply regularization to the other components, since it may improve convergence properties, as we will show in Section V. The dual problem is given by
\[
\max_{\lambda_k} g(\{\lambda_k\}) = d^*,
\]
(17)
subject to (14b) and (14c), where \( d^* \) is defined as the dual optimal solution. The dual problem (17) gives a lower bound on the primal optimal value \( p^* \) of problem (14), i.e.,
\[
d^* \leq p^*.
\]
(18)
The dual problem equals the primal problem, i.e. \( d^* = p^* \), if problem (14) is convex and the constraints satisfy Slater’s constraint qualifications [19]. As we have formed a convex SQP subproblem (14) and assume that the Slater’s constraint qualifications are satisfied for this problem formulation, we have that \( d^* = p^* \) in our SQP and dual decomposition approach. We maximize the dual problem (17) with a steepest ascent method, i.e.,
\[
\lambda_k^{i+1} = \max \{0, \lambda_k^i + \rho_k \left( \sum_{m \in \mathcal{M}} \tilde{A}_m \nu_{m,k}^i + \tilde{B}_m \nu_{m,k}^i + \tilde{E}_m \right) \}
\]
(19)
for all \( k \in \mathcal{K} \), where \( \rho_k \geq 0 \) is chosen small enough such that the dual problem converges. In our approach, note that in 1 iteration, we solve both the SQP subproblem (14) as well as update the Lagrangian multiplier \( \lambda_k \) for all \( k \in \mathcal{K} \). We have found that as long as \( R_m \) for all \( m \in \mathcal{M} \) are well chosen, this is the fastest way to let the dual problem converge. The dual problem (17) is terminated similarly to the SQP (4), i.e., it is terminated under the condition that
\[
\| J^{i+1} - J^i \| \leq \Delta_{\text{tol}},
\]
(20a)
where \( \Delta_{\text{tol}} \) is a certain specified tolerance, and
\[
J^i = \sum_{k \in \mathcal{K}} \left( \sum_{m \in \mathcal{M}} \max(0, \sum_{k \in \mathcal{K}} \tilde{A}_m \nu_{m,k}^i + \tilde{B}_m \nu_{m,k}^i) + \nu_1 \right)
\]
(20b)
where \( \nu_1 \geq 0 \) and \( \nu_2 \geq 0 \) are penalty parameters. The merit function (20b) has a cost term that defines the fuel consumption, a penalty term related to the violation of the state equality constraints (11c), and a penalty term related to the augmented inequality constraint (11f). Indeed, these two constraints are the only constraints that may be violated at iteration \( i \) of the dual problem (17), and thus have a contribution in the merit function (20b). This concludes the SQP and dual decomposition approach presented in this section, where by formulating (11) as a convex SQP subproblem (14), we could form the dual problem (17), which solves the vehicle energy management problem (11).

V. RESULTS

In this section, we show the results of two simulation studies where the Ecodriving problem is present. In the first simulation study, we will use the SQP approach presented in Section III to replicate the results of the Ecodriving problem for a full-electric vehicle (FEV) detailed in [12], using the SQP approach presented in Section II. In the second simulation study, we consider an energy management problem for a series-hybrid electric vehicle, and solve this with the approach presented in Section IV. The results of this simulation are comparable to the results presented in [15].
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yet not exactly the same, due to slightly different models that are used.

A. Full Electric Vehicle

The full electric vehicle simulation study as presented in [12] has a powertrain consisting of an electric motor (EM) and a high-voltage battery (HVB). However, in this case study, the HVB is not considered, i.e., it is assumed that the HVB has infinite energy storage and no power limitations, which makes it a simple but representative example for ecodriving. Furthermore, the optimal control problem given in [12] does not consider input and state constraints. This follows from the fact that [12] applies Pontryagin’s Maximum Principle, in which it is hard to introduce state and input constraints. We solve the ecodriving problem presented in [12] using the discretization approach as presented in Section II, with \( \tau = 0.0005 \), and SQP, as presented in Section III. We also show that with the SQP approach, adding state and input constraints becomes trivial.

The optimal control problem defined in [12] is given by the ecodriving problem (1) as presented in Section II, with

$$ g \sin(\alpha(s)) = p_0 + p_1 s + p_2 s^2 + p_3 s^3. $$

(21)

The parameters used for the FEV simulation study are given in Table I. We note that in this formulation, the control input \( u \) is a torque percentage defined as \( u = \frac{T}{T_{\text{max}}} \times 100\% \), in which \( T \) is the EM mechanical torque and \( T_{\text{max}} \) is the maximum EM torque. It is important to remark that the discretized objective function (2a) is not convex with the chosen parameters; however, choosing \( R_k = H_k \) in the SQP subproblem (4a) results in a convex objective function after the substitution of the state variables (9) into the SQP subproblem. To select initial conditions for the input and states, we choose a constant velocity profile \( v_k = 0 \) m/s for all \( k \in K \), and determine \( s_k \) and \( \mu_k \) for all \( k \in K \) from the longitudinal vehicle dynamics (2b). We set the tolerance to terminate the SQP algorithm defined in (5a) to \( \Delta \text{tol} = 1 \) and select for the penalty parameter \( \nu = 2 \times 10^3 \) in (5b). In Fig. 2, the results of the FEV simulation study, without state and input constraints are shown, in which we see that the initial and final state constraint are satisfied. We further note that the speed curve is not symmetrical, due to the road profile; close to where the slope is steepest, speed is maximized, as is expected. The cost as defined by the discrete-time objective function by the merit function (5b) is \( J = 1227247.8 \), which differs by 0.1% to the cost obtained in [12]. This small difference may be caused due to numerical inaccuracies presented in both this work, as well as in [12], and the finite sampling time. It is interesting to note that although global optimality is guaranteed neither in the approach used in this paper nor in the approach used in [12], the solutions obtained are practically identical. We remark that global optimality is guaranteed in neither our approach in this paper, nor the approach based on PMP in [12], because both approaches only consider necessary conditions for optimality. As an illustration to the ease of adding state and input constraints, in Fig. 3, the results of the simulation study with a maximum speed constraint of 12 m/s are shown. We observe that with a cost of \( J = 1511796.5 \), by adding this constraint, the cost becomes higher than without state constraints, as is expected.

B. Series-Hybrid Electric Vehicle

In this simulation study, we consider the Series-Hybrid Electric Vehicle (SEHV) case study presented in Section IV. We solve the case study in a ‘forward’ and ‘backward’ simulation using the approach presented in Section IV. The term ‘forward’ and ‘backward’ refer to way in which the longitudinal vehicle dynamics differential equations (1c) are treated. That is, in ‘backward’ simulation, the differential equations are treated as ‘quasi-static’ equations, i.e., the states \( v \) and \( s \) are given a priori and in ‘forward’ simulation, the differential equations are treated ‘dynamically’, i.e., the states remain decision variables. We will refer to ‘forward’ optimization as solving the vehicle energy management problem with ecodriving (11), and refer to ‘backward’ optimization as solving (11) as an energy management problem without ecodriving, where the vehicle trajectory information, i.e.

<table>
<thead>
<tr>
<th>Table I: Full Electric Vehicle Parameters</th>
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<tbody>
<tr>
<td>( p_0 = 3 )</td>
</tr>
<tr>
<td>( p_1 = 0.4 )</td>
</tr>
<tr>
<td>( p_2 = -1 )</td>
</tr>
<tr>
<td>( p_3 = 0.1 )</td>
</tr>
</tbody>
</table>
speed \(v_k\) and distance \(s_k\) for all \(k \in \mathcal{K}\), are given. We further solve the ‘forward’ optimization problem with the SQP (14), although without applying dual decomposition, which leads to a Quadratically Constrained Quadratic Program (QCQP), which we will refer to as the ‘direct’ method, to validate the dual decomposition method. We refer to the SQP and dual decomposition approach presented in Section IV as the ‘distributed’ method. To show additional capabilities of the SQP approach, we show an example where we introduce time-varying minimum and maximum speed constraints.

We base our case study on the work done in [15], in which a convex optimization approach is taken to solve the SEHV case study, where it is formulated a second-order cone program. Due to the chosen problem formulation, the authors in [15] have opted for a piece-wise linear EM model, linear EGU model and a quadratic HVB model. Furthermore, the authors in [15] define the optimization problem in the space domain, for which the physical interpretation of some parts of their problem formulation is not easy to understand. As we have defined the SEHV case study in the time domain, we do not face this issue. To have somewhat comparable results, we fit the parameters of our quadratic EM model using a least-squares fitting tool. We further approximate the linear EGU model with a quadratic EGU model by choosing the quadratic coefficient \(\gamma_{\text{egu}}\) in (12c) sufficiently small. The parameters used for the simulation study are shown in Table II. We remark here that the upper and lower bounds for all \(k \in \mathcal{K}\) are equal to zero.

The simulations are done for 1080 s over a distance of 21 km with a step size \(\tau = 5\) s, which gives an optimization horizon \(K = 216\). In ‘backward’ optimization, the speed is given by a constant speed of \(v_k = 70\) km/h for all \(k \in \mathcal{K}\), and in ‘forward’ optimization the initial and final velocity are also has a convex quadratic objective function and linear constraints. As the EGU dual problem \(\phi_{\text{egu}}\) also has a convex quadratic objective function and linear constraint, it may be solved as a QP problem, and thus we can see that the results for ‘forward’ optimization using the ‘direct’ method are given. It can be seen that the results for ‘forward’ optimization using the ‘direct’ method and ‘distributed’ method are almost the same, which validates the dual decomposition approach.

In the ‘forward’ optimization results, we see that between 0 and 2 km, where the slope becomes negative, the vehicle reaches the lower speed bound. This action allows the available potential energy from the road profile, between 2 and 13 km, to be maximally converted to kinetic energy; the vehicle speed is maximized in this interval. After 13 km, the road gradient becomes positive and speed is minimized such that the final speed constraint is met. Therefore, we can see that as a result of having the speed as a decision variable, in the ‘forward’ case, the EGU may provide less power over the course of the trajectory, and noticeably less braking power is applied, when it is compared to the ‘backward’ case. The fuel consumption of the ‘backward’ and ‘forward’ simulation cases are 23.41 l/100 km and 22.31 l/100 km respectively. Thus, by including the ecodriving problem into the vehicle energy management problem, approximately 4.7% decrease

\[
R_{\text{egu}} = (1 - \alpha_R)H_{\text{egu}} + \alpha_R,
\]  

where \(\alpha_R\) is a tuning parameter and we note that \(\alpha_R = 0\) gives \(R_{\text{egu}} = H_{\text{egu}}\). The EM dual problem \(\phi_{\text{em}}\) has a non-convex quadratic objective function and nonlinear constraints, and thus we are restricted to choose an \(R_{\text{em}} \geq 0\). Specifically, we choose \(R_{\text{em}} = 0.02\|\nu\|^2\). As initial conditions for the ‘forward’ optimization, we choose the solutions of the ‘backward’ optimization. For the termination of the dual problem (17), we choose \(\delta_{\text{obj}} = 10^{-3}\) in (20a) and \(\nu_1 = 10^5\), \(\nu_2 = 10^4\) for the penalty parameters of the merit function. In Fig. 4, the simulation results for ‘backward’ and ‘forward’ optimization using the ‘distributed’ method, and ‘forward’ optimization using the ‘direct’ method are given. It can be seen that the results for ‘forward’ optimization using the ‘direct’ method and ‘distributed’ method are almost the same, which validates the dual decomposition approach. In the ‘forward’ optimization results, we see that between 0 and 2 km, where the slope becomes negative, the vehicle reaches the lower speed bound. This action allows the available potential energy from the road profile, between 2 and 13 km, to be maximally converted to kinetic energy; the vehicle speed is maximized in this interval. After 13 km, the road gradient becomes positive and speed is minimized such that the final speed constraint is met. Therefore, we can see that as a result of having the speed as a decision variable, in the ‘forward’ case, the EGU may provide less power over the course of the trajectory, and noticeably less braking power is applied, when it is compared to the ‘backward’ case. The fuel consumption of the ‘backward’ and ‘forward’ simulation cases are 23.41 l/100 km and 22.31 l/100 km respectively. Thus, by including the ecodriving problem into the vehicle energy management problem, approximately 4.7% decrease
in fuel consumption is achieved. This is compared to the fuel consumption obtained in [15], which are 24.35 l/100 km and 23.98 l/100 km for the ‘backward’ and ‘forward’ case respectively. This is a 1.54 % decrease in fuel consumption between the ‘backward’ and ‘forward’ case. We may explain this difference in fuel consumption savings largely due to the different models used.

Finally, the convergence of the SQP + dual decomposition approach presented in section IV is shown in Fig. 5. We see that the number of iterations for the dual problem (17) is greatly reduced by tuning $R_{eq}$, even though $R_{eq} = H_{eq} = 2 \times 10^{-5}$ yields a convex problem. Specifically, the dual problem (17) converges after 1003 iterations with $\alpha_R = 0.002$ and converges after around 21000 iterations with $\alpha_R = 0$. We refer to convergence as the point where the dual problem may be terminated according to the tolerance specified by (20a), although we have not terminated the dual problem at these specified iterations for the purpose of illustration. This shows the additional advantage of regularization, which is the potential to improve converge, whereas we have used regularization for the EM dual problem $q_{eq}$ to enforce convexity. However, from Fig. 5 we also see that we sacrifice monotonicity for speed of convergence by choosing $\alpha_R > 0$, where the graphs shows more oscillatory behavior as $\alpha_R$ increases. However, this is not an issue as long as the required tolerances are reached for the termination of the dual problem.

VI. CONCLUSION

In this paper, we have solved the ecodriving problem using a Sequential Quadratic Programming (SQP) algorithm. We have formulated the ecodriving problem as a discrete-time nonlinear optimal control problem, and have shown that due to the discretization in this approach, the formulation may be nonconvex. In the SQP algorithm, we have formed a convex SQP subproblem using Thikonov regularization. To showcase the scalability of the distributed optimization approach presented in [1], we have embedded the SQP algorithm into this distributed optimization approach, which allows it to be used for Complete Vehicle Energy Management (CVEM) in combination with ecodriving. We have done so by formulating a vehicle energy management with ecodriving problem as a convex SQP problem and applying dual decomposition to solve a dual problem. We have considered two case studies for the ecodriving problem. In the first case study, we have solved the ecodriving problem for a Full Electric Vehicle using the SQP algorithm and have shown that the algorithm yields a very similar result as the benchmark problem set in [12], and shown that adding state constraints is trivial using SQP. In the second case study, we have solved an energy management problem with ecodriving for a Series-Hybrid Electric Vehicle with the SQP algorithm and dual decomposition. We have shown that using our approach, we have had the opportunity to use more representable powertrain models than in [15], and as a result have the potential to save much more fuel by incorporating ecodriving to the energy management problem.

REFERENCES

Eco-Driving for Energy Efficient Cornering of Electric Vehicles in Urban Scenarios

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Abstract: In this paper, we propose a model for eco-driving that considers cornering effects. The proposed model purely relies on the geometric configuration of the vehicle and road. Consequently, we propose and eco-driving optimal control problem formulation that is suitable for both straight and curved trajectories in urban scenarios. Moreover, it can be applied for vehicles with front wheel drive (FWD) or rear wheel drive (RWD). We use a case study for an electric vehicle executing cornering maneuvers to validate the proposed approach and highlight its advantages with respect to traditional eco-driving formulations. Results show an approximated improvement of 8% in energy savings with respect to traditional eco-driving strategies, especially in trajectories with large curvatures.

Keywords: Energy management system, optimal control, velocity control, electric vehicles, vehicle dynamics.

1. INTRODUCTION

In recent years, electrification of transport systems or electromobility (Grauers et al., 2013), has been raised as a promising approach to mitigate environmental effects caused by CO2 emissions and to face the imminent depletion of fossil fuels reserves. However, the goal of complete acceptance of electromobility in the market is still far and several problems like range anxiety still have to be solved. Range anxiety is defined as the fear experienced by drivers of having insufficient energy to arrive to the next charging station (AccessScience Editors, 2014). In general, energy management strategies are a direct method to reduce range anxiety. In particular, the capability to extend driving range by reducing power demand demonstrated by eco-driving techniques has positioned these approaches as strong tools alleviate effects of range anxiety, e.g., Flores et al. (2019).

As a general concept, eco-driving is a selection of energy efficient driving strategies (Bingham et al., 2012), which can be obtained by solving an optimal control problem (OCP) that aims to find optimized velocity profiles that reduce the total consumption of the vehicle, e.g., see Sciarretta et al. (2015). This concept has been studied in terms of the energy source of the vehicle, e.g., for conventional vehicles in Maamria et al. (2017), for hybrid electric vehicles by Khalik et al. (2018), and for electric vehicles by Petit and Sciarretta (2011). Eco-driving has also been analysed in terms of the surrounding operational conditions. For urban scenario cases, Nunzio et al. (2013) have included traffic light information into the eco-driving formulation, Henriksson et al. (2017) use traffic statistic data to create velocity corridors where solutions the eco-driving OCP lay, and Han et al. (2018) consider the effects of the preceding vehicle in the problem formulation. Interestingly, the literature about eco-driving for urban trajectories where cornering is considered is scarce.

In (Beckers et al., 2019), a high-fidelity model is presented to calculate additional energy losses during cornering of the vehicle. The results presented in this work have shown that energy losses during cornering maneuvers should not be neglected. Cornering losses become relevant during urban scenarios where the vehicle might follow routes where several turning maneuvers are performed. Often, eco-driving formulations that consider the road curvature include a constraint linked to the centripetal acceleration, which indirectly limits the maximum velocity during cornering, e.g., Polterauer et al. (2019). In this case, the cornering losses are not directly considered in the formulation. As a result, the solutions obtained are unlikely to be energy optimal during the cornering maneuver. An interesting case is presented in Ikezawa et al. (2017), where a dynamical vehicle model is used to describe a simplified model that can be used to formulate a eco-driving OCP. Unfortunately, this description depends on cornering stiffness of the tires, which can show significant variations between vehicles and road conditions. Other approaches aim to achieve energy efficient cornering by applying energy efficient torque vectoring to the vehicle (Edrén et al., 2019). Unfortunately, these approaches partially neglect the eco-driving concept by only focusing on the cornering maneuver itself.

In this paper, we aim to bridge the gap observed in literature by extending the current eco-driving OCP formulation of Padilla et al. (2018) to consider cornering effects. In particular, our main contributions are the use of a kinematic bicycle model to approximate the dissipative forces produced during cornering in the longitudinal direction of
the vehicle. This model purely depends on the geometry of the road and the vehicle, which makes it suitable to be deployed in real applications where the tire properties are often unknown. Thus, we propose a trajectory dependent model for eco-driving that can be used for vehicles with front wheel drive (FWD) and rear wheel drive (RWD). This allows us to formulate generalized eco-driving OCP that is suitable to be used in urban routes.

The remainder of this paper is organized as follows. In Section 2, the cornering effects are approximated in a low complexity model for eco-driving applications. Section 3, proposes and OCP formulation that takes advantages of the modeling choice proposed in this paper. Later, a case study for an electric vehicle performing a cornering maneuver in an urban intersection is presented in Section 4. Here we will validate our approach using a high-fidelity model to calculate losses during cornering and we will highlight the advantages of the proposed approach. Finally, we will draw conclusions in Section 5.

2. A TRAJECTORY-DEPENDENT MODEL FOR ECO-DRIVING

In this section, we aim to provide a general vehicle dynamical model to be used in the eco-driving optimal control problem proposed in this paper. To this end, we analyse the representations used for straight trajectories, which coincides with traditional models used for eco-driving in the current literature. Later, we propose a generalized description of the dissipative forces for curved trajectories, which are often present in urban scenarios. Finally, we show that the generalized models obtained for curved trajectories can easily describe the dynamics for straight trajectories as well.

The main objective of eco-driving strategies is to obtain energy optimal velocity profiles. In general, the dynamical models considered in traditional eco-driving approaches represent the longitudinal vehicle dynamics by the interaction between the traction force in the longitudinal axis $F_t(t)$ and dissipative forces $F_d(t)$, i.e.,

$$ma = F_t - F_d,$$  

where $m$ represents the equivalent mass of the vehicle, and $a(t) = \frac{dv}{dt}$ is the vehicle acceleration. The definition of $F_d(t)$ in (1) can vary depending on the trajectory that the vehicle is describing. Specifically, we consider the cases for straight and curved trajectories below.

2.1 Straight Trajectories

A traditional assumption for eco-driving problem formulations is that the vehicle moves in a straight trajectory such that the force produced by aerodynamics is

$$F_{air}(v, s) = \frac{1}{2} \rho \pi v^2 A_f (c_s \cos(\alpha(s)) + \sin(\alpha(s))).$$

where $v(t)$ represents the vehicle velocity, $s(t)$ describes displacement, $\alpha(s(t))$ is the road grade, $g$ is the gravitational constant, $c_s > 0$ is the rolling resistance, and $\sigma_d = \frac{1}{2} \rho_A v T$ with the aerodynamical drag coefficient $c_d > 0$, air density $\rho_A$ and frontal area of the vehicle $A_f$.

In the right-hand side of (2), the term $F_{air}$ represents the force produced by aerodynamical drag, and the term $\eta$ the road friction, and $F_{friction}$ the friction force.

2.2 Curved Trajectories

The main contribution of this section is the inclusion of cornering effects in the model used for eco-driving OCP in urban scenarios, i.e., assuming curved trajectories. The effect of cornering in terms of energy consumption has been discussed in Beckers et al. (2019), where a highly detailed non-linear model has been developed to describe additional tire slip losses during cornering. The results presented in the aforementioned work show that the energy consumed as consequence of cornering is significant. In this section, we follow a simplified approach to approximate the effects of cornering into a low-complexity model that will be used to formulate the eco-driving OCP in Section 3.

Let us consider the curved trajectory observed in Fig. 1, which is characterized by a position-dependent curvature function $R(s)$, where the curvature is defined as the reciprocal of the radius $R(s)$, i.e.,

$$K(s) = \frac{1}{R(s)}.$$  

The radius $R$ is measured from the instantaneous center of rotation (ICR) to the vehicle center of gravity (CG). We
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Electric Vehicle Enhanced Range, Lifetime And Safety Through INGenious battery management

In order to analyse the interaction of traction and dissipative forces in this case, we use a kinematic bicycle model. Despite the simplicity of a kinematic bicycle model, it can achieve similar results as a dynamical bicycle model for vehicle control purposes (Kong et al., 2015). The kinematic bicycle model considers that the two front and rear wheels are respectively lumped into one single front and rear wheel, which in Fig. 1 are represented in red color. Even though this is strictly true only for low lateral vehicle accelerations, applying this model to high-velocity corners will result in an over-estimate of the cornering energy, thereby making the model quantitatively conservative. Moreover, we assume vehicles with front-wheels-only steering systems, implying that the rear wheel will be aligned with the longitudinal axis of the vehicle for the entire route. For the sake of simplicity, we assume that the dissipative forces $F_{d,rl}$ and $F_{d,ru}$ are aligned with the longitudinal axis of the vehicle. Note that the velocity of the vehicle $v(t)$ is tangential to the trajectory and shows an angle $\beta(s(t)) \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ with respect to the longitudinal axis, which is given by

$$\beta(s) = \arcsin(l_c K(s)) \quad (5)$$

The total centripetal force applied at CG and its projection into the longitudinal vehicle dynamics are given by

$$F_c(v, s) = m u^2 K(s), \quad (6)$$

and

$$F_{cs}(v, s) = F_c(v, s) \cos(\frac{\pi}{2} - \beta(s(t))) = m u^2 K(s)^2, \quad (7)$$

respectively.

An analysis of the forces acting on the longitudinal axis of the vehicle shows that for curved trajectories the total dissipative force in that direction is given by

$$F_d(v, s) = F_{d,rl} + F_{d,ru} + F_{di} \quad (8)$$

Note that projection of the centripetal force into the longitudinal axis (7) can be physically interpreted as a lumped approximation in the longitudinal direction of the lateral forces generated on the tires during cornering. From (2) and (7), it is possible to rewrite (8) as

$$F_d(v, s) = mg(c_0 \cos(\alpha(s)) + \sin(\alpha(s))) + (\sigma_d + m d K(s)^2)u^2. \quad (9)$$

Unlike to the approach of Iezawa et al. (2017), the dissipating force introduced by cornering $F_{di}$ is an approximation that only depends on the geometric configuration of the vehicle and the road, which comes as a consequence of the use of a kinematic model. In Section 4, we will use a numerical example to show that albeit the simplicity of the obtained approximation, it properly captures the behavior of the vehicle during cornering for control purposes.

The traction force on the longitudinal direction $F_t$ depends on the drive configuration of the vehicle, i.e., in a RWD configuration the total motor traction force $F_m(t)$ is directly applied directly to the longitudinal direction of the vehicle, while for FWD case only a fraction of $F_m(t)$ is applied in the longitudinal direction. This idea is depicted in Fig. 2, where the traction forces for the two drive configurations are shown. It is clear to see that the traction force for a vehicle with RWD can be described by (3). On the other hand, the traction force in the longitudinal direction for a FWD vehicle depends on the steering angle as

$$F_t(F_u, s) = F_u \cos(\beta(s)), \quad (10)$$

where according to the kinematic bicycle model

$$\beta(s) = \arctan\left(\frac{l_r + l_l}{2l} \tan(\beta(s))\right). \quad (11)$$

After substituting (5) into (11) and applying a composition of trigonometric and inverse trigonometric functions, the traction force into the longitudinal direction (12) can be reformulated as

$$F_t(F_u, s) = \frac{F_u}{1 + \left(\frac{u^2 K(s)^2}{F_u^2} + 1\right)} \quad (12)$$

The projection of the centripetal force and the traction force in the direction of motion allows us to generalize the longitudinal vehicle dynamics (1) model for curved trajectories as

$$m a = F_t(F_u, s) - F_{di}(v, s), \quad (13)$$

where $a(t) = \frac{dv}{dt}$ is the acceleration in the tangential direction to the trajectory. Considering that the forces are analyzed in the longitudinal axis of the vehicle, the use of $a(t)$ instead of the longitudinal acceleration $a_l(t)$ in (13) is justified by a small angle approximation such that $a(t) = a_l(t) \cos(\beta(s(t))) \approx a_l(t).$ Additionally in (13), $F_d(v, s)$ is given by (9), and $F_t(F_u, s)$ is described by (3) and (12) for vehicles with FWD and RWD, respectively.

As a final observation, it should be noted that taking $K(s) = 0$ implies that the vehicle moves on a straight trajectory. For this specific case, (9) is equivalent to (2), which represent the dissipating forces for a straight trajectory. Similarly, for FWD vehicles (12) is equivalent to (3), which describes the traction force in the longitudinal direction for vehicles driving in straight trajectories. These observations show the generality of the proposed models.

3. OPTIMAL CONTROL PROBLEM

A continuous-time optimal control problem (OCP) formulation that represents the eco-driving problem for urban city scenarios is provided in this section. In particular, we will extend the eco-driving formulation proposed in Padilla et al. (2018) to include the cornering effects captured by the trajectory dependent models presented in the previous section. Additionally, we discuss the differences between the OCP proposed in this paper and common approaches of eco-driving for cornering available in literature.
3.1 Problem Formulation

For a route with given curvature \( K(s) \) and road grade \( \alpha(s) \), eco-driving aims to minimize the aggregative power \( P(v(t), F_a(t)) \) over a fixed period of time \([t_0, t_f]\) required by a vehicle driving a trajectory \( s(t) \in [s_0, s_f] \), while being subject to position dependent velocity and acceleration bounds \( v(t) \in [v_{min}, v_{max}] \), \( a(t) \in [a_{min}, a_{max}] \), respectively. Moreover, the vehicle longitudinal dynamics and initial final conditions for position and velocity are considered. A mathematical formulation of the eco-driving problem as an OCP is given by

\[
\min_{s(t), v(t), a(t), F_a(t)} \int_{t_0}^{t_f} P(v(t), F_a(t))dt \quad (14a)
\]

subject to \( ma(t) = F_a(t) - F_d(v(t), s(t)) \), \( 14b \)
\[ \frac{dv}{dt} = v(t), \quad 14c \]
\[ \frac{ds}{dt} = s(t), \quad 14d \]
\[ v(t_0) = v_0, \quad v(t_f) = v_f, \quad 14e \]
\[ a(t_0) = a_0, \quad a(t_f) = a_{-0.2g}, \quad 14f \]
\[ a(t)^2 + v(t)^2 K(s(t))^2 \leq (\mu_s g)^2, \quad 14g \]
\[ v(t) \leq v_{max} \leq \pi(t) \leq \pi(s(t)), \quad 14h \]
\[ a(t) \leq a_{max} \leq a(t) \leq a(s(t)), \quad 14i \]

where the power consumed by the electric motor and driveline at a given time instant is

\[ P(v, u) = \beta_2 F_a^2 + \beta_1 v F_a + \beta_0 u^2 \]

with positive coefficients \( \beta_2 \) to penalize the Ohmic losses, \( \beta_1 \) to describe effective power consumed, and \( \beta_0 \) that penalizes the friction losses in the electric motor. The longitudinal vehicle dynamics are described by (14b) with the definitions provided for (13), the time evolution of acceleration, velocity and position are described by (14d) and (14c), and boundaries for position of velocity are represented by (14e) and (14f), respectively. Finally, (14g) represents a constraint imposed on the total acceleration of the vehicle, where \( s_{max} > 0 \) is a friction coefficient dependent on the characteristics of the tires and the road conditions. In order to give a physical justification to (14g), let us note that the maximum friction force between the road and tires is given by \( F_{fric} = \mu_s m g \). For safety reasons, is required to avoid the vehicle to slip, which implies that the total force applied to the vehicle is lower than the maximum friction force during normal operation, i.e.,

\[ (ma(t))^2 + (mv(t)^2 K(s(t)))^2 \leq (\mu_s g)^2, \quad (16) \]

which can be simplified into (14g). The first term in the left hand side of (16) represents the resultant force applied to the vehicle in the tangential direction of the trajectory, and the second term is the centripetal force. Note that (14g) is mainly relevant during cornering. During straight trajectories this constraint is inactive because upper acceleration bound in (14i) is expected to satisfy \( \pi(s) \leq \mu_s g \).

3.2 Effects of Cornering

From the scarce literature about eco-driving approaches that consider cornering effects, it is possible to note that often the OCP formulation includes a constraint linked to the centripetal acceleration of the vehicle, see, e.g., Polterauer et al. (2019), which is similar to (14g). This type of hard constraint implicitly imposes a limit to the maximum velocity during cornering. In the approach presented in this paper, we improve the representation of cornering effects by including those effects in longitudinal vehicle dynamics, i.e., (14b). This specific modeling choice can be seen as a soft constraint that highly penalizes the velocity while cornering. To this end, we will use a simplified example that, without losing generality, allow us to observe the effects cornering in the OCP (14).

Let us consider a RWD vehicle, driving in a circular trajectory on a flat road, i.e., \( K(s) = \frac{1}{R} \) and \( \alpha(s) = 0 \). It is possible to find an equivalent formulation to the OCP (14) by substituting (14b) into (14a) (see, e.g., (Boyd and Vandenberghe, 2004, §4.1.1)), leading to

\[
\min_{s(t), v(t), a(t)} \int_{t_0}^{t_f} P(v(t), ma(t) + F_d(v(t), s(t)))dt \quad (17)
\]

subject to (14c)-(14i), in which

\[ P(v, ma + F_d(v, s)) = \beta_2 (m(a + g c) + (s a + m l K^2))^2 \]

\[ + \beta_3 (m(a + g c) + (s a + m l K^2))^2 + \beta_0 u^2 \quad (18) \]

Thus, we can note the the term \( ml K^2 \) drastically penalizes velocity in the new cost function. This penalization, depends quadratically on the road curvature, which implies that the optimal solution might show lower decelerations during cornering maneuvers.

4. CASE STUDY

In this section, we study an electric vehicle executing a cornering maneuver in an urban environment with different curvatures. The advantages of the model and OCP formulation presented in this paper are highlighted in this example. To this end, we will contrast the results with traditional eco-driving approaches. Moreover, we will validate the proposed approach using a high-fidelity model to calculate the instantaneous power and total energy consumption produced during the cornering maneuvers analyzed.

Let us consider an electric vehicle with RWD and parameters listed in Table 1. The vehicle executes a cornering maneuver on a city intersection as is depicted in Fig. 3, which curvature is defined as

\[ K(s) = \frac{1}{R} \quad \text{for} \quad s_0 \leq s \leq s_f, \quad (19) \]

Table 1. Vehicle and OCP parameters

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m )</td>
<td>13600</td>
<td>[kg]</td>
</tr>
<tr>
<td>( \sigma_d )</td>
<td>1.24625</td>
<td>[N/m²]</td>
</tr>
<tr>
<td>( c_v )</td>
<td>0.007</td>
<td>-</td>
</tr>
<tr>
<td>( \mu_s )</td>
<td>0.35</td>
<td>-</td>
</tr>
<tr>
<td>( \beta_0 )</td>
<td>0.292</td>
<td>[w/m²]</td>
</tr>
<tr>
<td>( \beta_1 )</td>
<td>1.005</td>
<td>-</td>
</tr>
<tr>
<td>( \beta_2 )</td>
<td>2.652x4</td>
<td>[w/m²]</td>
</tr>
<tr>
<td>( v_0 )</td>
<td>0</td>
<td>[m]</td>
</tr>
<tr>
<td>( s_0 )</td>
<td>150</td>
<td>[m]</td>
</tr>
<tr>
<td>( v_1 )</td>
<td>30</td>
<td>[m/s]</td>
</tr>
<tr>
<td>( v_f )</td>
<td>35</td>
<td>[m/s]</td>
</tr>
<tr>
<td>( \pi )</td>
<td>60</td>
<td>[m/s]</td>
</tr>
<tr>
<td>( a_{max} )</td>
<td>0</td>
<td>[m/s²]</td>
</tr>
<tr>
<td>( a_{-0.2g} )</td>
<td>0.2g</td>
<td>[m/s²]</td>
</tr>
</tbody>
</table>

| \( a_{max} \) | -0.2g | [m/s²] |
Fig. 3. Urban intersection considered in the case study.

where $s_c^0$ is the position where the corner begins and $s_c^f$ is the final corner position. The curvature (19), will be specified for three different scenarios that are detailed in Table 2.

The OCP (14) is formulated using the parameters in Table 1. To find the solutions of this OCP, we first discretize the problem (14) and later apply a static optimization technique, i.e., sequential quadratic programing (Bogg and Tolle (1995)). The optimal solutions to (14) are depicted in Fig. 4 as velocity and acceleration profiles for each scenario considered in the case study. Note that the vertical lines depicted in both profiles indicates the initial and final positions of the curved section of the trajectory. It can be observed that the vehicle decelerates before entering the curved section of the trajectory. When the vehicle enters the curved section of the trajectory the deceleration rate is reduced and approximately at half of the curved section the vehicle begins accelerating. As soon as the vehicle leaves the curved section of the trajectory the acceleration rate is immediately increased. Interestingly, the differences between scenarios considered is clearly observed in the velocity profiles, where at higher curvatures lower velocities are observed in the curved section of the trajectory. This results is expected because velocity is highly penalized for high curvatures in the road. This effect has been discussed in Section 3.2.

In order to highlight the advantages of our general approach, we compare the optimal solution to the OCP (14) with a traditional eco-driving OCP formulation. Specifically, a traditional OCP considers the constraint (14g), but the cornering effects in (14b) are neglected. In Fig. 5, the optimal velocity and acceleration profiles for traditional and general eco-driving formulations are presented for the specific scenario where $R = 14 \text{ [m]}$. Note that the velocity profile of the traditional approach shows constant velocity during the curved section of the road, which also indicates zero acceleration during that section. This is expected since traditional eco-driving strategies avoids changes in acceleration to reduce energy consumption. As a consequence, the vehicle crosses the curved section with the maximum possible velocity, which is defined by constrain (14g). On the other hand, the optimal strategy obtained

![Diagram](image-url)

**Table 2. Parameters for different scenarios.**

<table>
<thead>
<tr>
<th>$R$</th>
<th>$s_c^0$</th>
<th>$70 + \frac{R\pi}{2}$</th>
<th>$s_c^f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>12 m</td>
<td>70 m</td>
<td>70 + 12$\pi$/2 [m]</td>
<td>$s_c^f$</td>
</tr>
<tr>
<td>14 m</td>
<td>70 m</td>
<td>70 + 14$\pi$/2 [m]</td>
<td>$s_c^f$</td>
</tr>
<tr>
<td>17 m</td>
<td>70 m</td>
<td>70 + 17$\pi$/2 [m]</td>
<td>$s_c^f$</td>
</tr>
</tbody>
</table>

![Diagram](image-url)
Table 3. Comparison of energy consumption between strategies.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Total Energy (Traditional)</th>
<th>Total Energy (General)</th>
<th>Diff. %</th>
</tr>
</thead>
<tbody>
<tr>
<td>R = 12 m</td>
<td>820.2 [kJ]</td>
<td>757.6 [kJ]</td>
<td>8.26</td>
</tr>
<tr>
<td>R = 14 m</td>
<td>697.5 [kJ]</td>
<td>658.2 [kJ]</td>
<td>4.45</td>
</tr>
<tr>
<td>R = 17 m</td>
<td>574.2 [kJ]</td>
<td>534.4 [kJ]</td>
<td>0.14</td>
</tr>
</tbody>
</table>

Fig. 7. Cumulative energy consumption for different scenarios and eco-driving strategies.

5. CONCLUSIONS

A model that approximates cornering forces into the longitudinal axis of the vehicle has been proposed in this paper. The simplicity of this model relies on the geometry of the vehicle and the road, which makes it easy to use for a broad spectrum of cases without the necessity of identification of specific tire parameters. Based on this model, we have proposed a general eco-driving optimal control problem formulation that can be used for both straight and curved trajectories. Moreover, it can be used for front wheel drive and rear wheel drive configurations of the vehicle. The advantages of the proposed OCP have been analysed on a case study, where the use of a high-fidelity model have allowed not only to validate the proposed model and OCP formulation, but also to show the improvements of this approach with respect to traditional approaches found in current literature. The results have shown that the use of the general OCP formulation proposed in this paper yields to an improvement energy savings for cornering maneuvers in trajectories with large curvatures, while for roads with small curvature the total consumption of the general strategy tends to approximate to the traditional approach. This is a consistent result since our OCP formulation resembles traditional eco-driving formulations when roads with no curvature are considered, as has been discussed in Section 2.2. Remarkably, in Fig. 7, it is possible to see that a higher energy efficiency of the proposed strategy mainly comes from a reduction of the energy losses in the electric machine.

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**D3.5 – Report on driving range prediction and extension algorithm**

**Author:** Paul Padilla (TU/e)

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